TABLE 5-17   Mass-Transfer Correlations for a Single Flat Plate or Disk—Transfer to or from Plate to Fluid

Situation	Correlation	Comments E = Empirical, S = Semiempirical, T = Theoretical	References[a]
A. Laminar, local, flat plate, forced flow	$N_{Sh,x} = \dfrac{k'x}{D} = 0.323(N_{Re,x})^{1/2}(N_{Sc})^{1/3}$ Coefficient 0.332 is a better fit.	[T] Low M.T. rates. Low mass-flux, constant property systems. $N_{Sh,x}$ is local k. Use with arithmetic difference in concentration. Coefficient 0.323 is Blasius' approximate solution.	[77] p. 183 [87] p. 526 [138] p. 79 [140] p. 518
		$N_{Re,x} = \dfrac{xu_\infty\rho}{\mu}$, x = length along plate	[141] p. 110
Laminar, average, flat plate, forced flow	$N_{Sh,avg} = \dfrac{k'_m L}{D} = 0.646(N_{Re,L})^{1/2}(N_{Sc})^{1/3}$ Coefficient 0.664 is a better fit. k'_m is mean mass-transfer coefficient for dilute systems.	$N_{Re,L} = \dfrac{Lu_\infty\rho}{\mu}$, 0.664 (Polhausen) is a better fit for $N_{Sc} > 0.6$, $N_{Re,x} < 3 \times 10^5$.	[91] p. 480
j-factors	$j_D = j_H = \dfrac{f}{2} = 0.664(N_{Re,L})^{-1/2}$	[S] Analogy. $N_{Sc} = 1.0$, f = drag coefficient. j_D is defined in terms of k'_m.	[141] p. 271
B. Laminar, local, flat plate, blowing or suction and forced flow	$N_{Sh,x} = \dfrac{k'x}{D} = (\text{Slope})_{y=0}\,(N_{Re,x})^{1/2}(N_{Sc})^{1/3}$	[T] Blowing is positive. Other conditions as above.	[77] p. 185
		$\dfrac{\dfrac{u_o}{u_\infty}\sqrt{N_{Re,x}}}{(\text{Slope})_{y=0}}$ $\begin{array}{ccccc} 0.6 & 0.5 & 0.25 & 0.0 & -2.5 \\ 0.01 & 0.06 & 0.17 & 0.332 & 1.64 \end{array}$	[140] p. 271
C. Laminar, local, flat plate, natural convection vertical plate	$N_{Sh,x} = \dfrac{k'x}{D} = 0.508 N_{Sc}^{1/2}(0.952 + N_{Sc})^{-1/4} N_{Gr}^{1/4}$ $N_{Gr} = \dfrac{gx^3}{(\mu/\rho)^2}\left(\dfrac{\rho_\infty}{\rho_0} - 1\right)$	[T] Low MT rates. Dilute systems, $\Delta\rho/\rho \ll 1$. $N_{Gr}N_{Sc} < 10^8$. Use with arithmetic concentration difference. x = length from plate bottom.	[141] p. 120
D. Laminar, stationary disk	$N_{Sh} = \dfrac{k'd_{disk}}{D} = \dfrac{8}{\pi}$	[T] Stagnant fluid. Use arithmetic concentration difference.	[138] p. 240
Laminar, spinning disk	$N_{Sh} = \dfrac{k'd_{disk}}{D} = 0.879 N_{Re}^{1/2} N_{Sc}^{1/3}$ $N_{Re} < \sim 10^4$	[T] Asymptotic solution for large N_{Sc}. $u = \omega d_{disk/2}$, ω = rotational speed, rad/s. Rotating disks are often used in electrochemical research.	[101] p. 60 [138] p. 240
E. Laminar, inclined, plate	$N_{Sh,avg} = 0.783 N_{Re,film}^{1/9} N_{Sc}^{1/3}\left(\dfrac{x^3\rho^2 g \sin\alpha}{\mu^2}\right)^{2/9}$ $N_{Re,film} = \dfrac{4Q\rho}{\mu^2} < 2000$ $N_{Sh,avg} = \dfrac{k'_m x}{D}$ $\delta_{film} = \left(\dfrac{3\mu Q}{w\rho g \sin\alpha}\right)^{1/3}$ = film thickness	[T] Constant-property liquid film with low mass-transfer rates. Use arithmetic concentration difference. Newtonian fluid. Solute does not penetrate past region of linear velocity profile. Differences between theory and experiment. w = width of plate, δ_f = film thickness, α = angle of inclination, x = distance from start soluble surface.	[141] p. 130 [138] p. 209
F. Turbulent, local flat plate, forced flow	$N_{Sh,x} = \dfrac{k'x}{D} = 0.0292 N_{Re,x}^{0.8}$	[S] Low mass-flux with constant property system. Use with arithmetic concentration difference. $N_{Sc} = 1.0$, $N_{Re,x} > 10^5$	[77] p. 191 [138] p. 201 [141] p. 221
Turbulent, average, flat plate, forced flow	$N_{Sh,avg} = \dfrac{k'L}{D} = 0.0365 N_{Re,L}^{0.8}$	Based on Prandtl's 1/7-power velocity law, $\dfrac{u}{u_\infty} = \left(\dfrac{y}{\delta}\right)^{1/7}$	
G. Laminar and turbulent, flat plate, forced flow	$j_D = j_H = \dfrac{f}{2} = 0.037 N_{Re,L}^{-0.2}$ $j_D = (k_C/G_M)N_{Sc}^{2/3}$ $j_H = (h'/C_p G)\,N_{Pr}^{2/3}$	[E] Chilton-Colburn analogies, $N_{Sc} = 1.0$, (gases), f = drag coefficient. Corresponds to item 5-17-F and refers to same conditions. $8000 < N_{Re} < 300,000$. Can apply analogy, $j_D = f/2$, to entire plate (including laminar portion) if average values are used.	[77] p. 193 [88] p. 112 [138] p. 201 [141] p. 271 [80] [53]

TABLE 5-17 Mass-Transfer Correlations for a Single Flat Plate or Disk—Transfer to or from Plate to Fluid (*Concluded*)

Situation	Correlation	Comments E = Empirical, S = Semiempirical, T = Theoretical	References[*]
H. Laminar and turbulent, flat plate, forced flow	$N_{Sh,avg} = 0.037 N_{Sc}^{1/3}(N_{Re,L}^{0.8} - 15{,}500)$ to $N_{Re,L} = 320{,}000$ $N_{Sh,avg} = 0.037 N_{Sc}^{1/3}$ $\times \left(N_{Re,L}^{0.8} - N_{Re,Cr}^{0.8} + \dfrac{0.664}{0.037} N_{Re,Cr}^{1/2} \right)$ in range 3×10^5 to 3×10^6.	[E] Use arithmetic concentration difference. $N_{Sh,avg} = \dfrac{k_m' L}{D}$, $N_{Sc} > 0.5$ Entrance effects are ignored. $N_{Re,Cr}$ is transition laminar to turbulent.	[88] p. 112 [138] p. 201
I. Turbulent, local flat plate, natural convection, vertical plate Turbulent, average, flat plate, natural convection, vertical plate	$N_{Sh,x} = \dfrac{k'x}{D} = 0.0299 N_{Gr}^{2/5} N_{Sc}^{7/15}$ $\times (1 + 0.494 N_{Sc}^{2/3})^{-2/5}$ $N_{Sh,avg} = 0.0249 N_{Gr}^{2/5} N_{Sc}^{7/15} \times (1 + 0.494 N_{Sc}^{2/3})^{-2/5}$ $N_{Gr} = \dfrac{gx^3}{(\mu/\rho)^2}\left(\dfrac{\rho_\infty}{\rho_0} - 1 \right)$, $\quad N_{Sh,avg} = \dfrac{k_m' L}{D}$	[S] Low solute concentration and low transfer rates. Use arithmetic concentration difference. $N_{Gr} > 10^{10}$ Assumes laminar boundary layer is small fraction of total.	[141] p. 225
J. Perforated flat disk Perforated vertical plate. Natural convection.	$N_{Sh} = 0.059 N_{Sc}^{0.35} N_{Gr}^{0.35} \left(\dfrac{d_h}{d} \right)^{0.04}$ Characteristic length = disk diameter d $N_{Sh} = 0.1 N_{Sc}^{1/3} N_{Gr}^{1/3}$ Characteristic length = L, electrode height	[E] $6 \times 10^9 < N_{Sc} N_{Gr} < 10^{12}$ and $1943 < N_{Sc} < 2168$ d_h = hole diameter [E] $1 \times 10^{10} < N_{Sc} N_{Gr} < 5 \times 10^{13}$ and $1939 < N_{Sc} < 2186$ Average deviation ± 10%	[162]
K. Turbulent, vertical plate	$N_{Sh,avg} = \dfrac{k_m' x}{D} = 0.327 N_{Re,\mathrm{film}}^{2/9} N_{Sc}^{1/3} \left(\dfrac{x^3 \rho^2 g}{\mu^2} \right)^{2/9}$ $\delta_{\mathrm{film}} = 0.172 \left(\dfrac{Q^2}{w^2 g} \right)^{1/3}$	[E] See 5-17-E for terms. $N_{Re,\mathrm{film}} = \dfrac{4Q\rho}{w\mu^2} > 2360$ Solute remains in laminar sublayer.	[141] p. 229
L. Cross-corrugated plate (turbulence promoter for membrane systems)	$N_{Sh} = c N_{Re}^{a} N_{Sc}^{1/3}$	[E] Entrance turbulent channel For parallel flow and corrugations: $N_{Sc} = 1483$, $a = 0.56$, $c = 0.268$ $N_{Sc} = 4997$, $a = 0.50$, $c = 0.395$ Corrugations perpendicular to flow: $N_{Sc} = 1483$, $a = 0.57$, $c = 0.368$ $N_{Sc} = 4997$, $a = 0.52$, $c = 0.487$	[134]
M. Turbulent, spinning disk	$N_{Sh} = \dfrac{k' d_{\mathrm{disk}}}{D} = 5.6 N_{Re}^{1.1} N_{Sc}^{1/3}$ $6 \times 10^5 < N_{Re} < 2 \times 10^6$ $120 < N_{Sc} < 1200$	[E] Use arithmetic concentration difference. $u = \omega d_{\mathrm{disk}}/2$ where ω = rotational speed, radians/s. $N_{Re} = \rho \omega d^2 / 2\mu$.	[55] [138] p. 241
N. Mass transfer to a flat plate membrane in a stirred vessel	$N_{Sh} = \dfrac{k' d_{\mathrm{tank}}}{D} = a N_{Re}^{b} N_{Sc}^{c}$ a depends on system. $a = 0.0443$ [40]; b is often 0.65–0.70 [89]. If $N_{Re} = \dfrac{\omega d_{\mathrm{tank}}^2 \rho}{\mu}$	[E] Use arithmetic concentration difference. ω = stirrer speed, radians/s. Useful for laboratory dialysis, R.O., U.F., and microfiltration systems. $b = 0.785$ [40]. c is often 0.33 but other values have been reported [89].	[40] [89] p. 965
O. Spiral type RO (seawater desalination)	$N_{Sh} = 0.210 N_{Re}^{2/3} N_{Sc}^{1/4}$ Or with slightly larger error, $N_{Sh} = 0.080 N_{Re}^{0.875} N_{Sc}^{1/4}$	[E] Polyamide membrane. $p = 6.5$ MPa and TDS rejection = 99.8%. Recovery ratio 40%.	[148]

[*]See the beginning of the "Mass Transfer" subsection for references.

TABLE 5-18   Mass-Transfer Correlations for Falling Films with a Free Surface in Wetted Wall Columns—Transfer between Gas and Liquid

Situation	Correlation	Comments E = Empirical, S = Semiempirical, T = Theoretical	References[*]
A. Laminar, vertical wetted wall column	$N_{Sh,avg} = \dfrac{k'_m x}{D} \approx 3.41\,\dfrac{x}{\delta_{film}}$ (first term of infinite series) $\delta_{film} = \left(\dfrac{3\mu Q}{w\rho g}\right)^{1/3} = $ film thickness $N_{Re,film} = \dfrac{4Q\rho}{w\mu} < 20$	[T] Low rates M.T. Use with log mean concentration difference. Parabolic velocity distribution in films. w = film width (circumference in column) Derived for flat plates, used for tubes if $r_{tube}\left(\dfrac{\rho g}{2\sigma}\right)^{1/2} > 3.0$. σ = surface tension If $N_{Re,film} > 20$, surface waves and rates increase. An approximate solution $D_{apparent}$ can be used. Ripples are suppressed with a wetting agent good to $N_{Re} = 1200$.	[138] p. 78 [141] p. 137 [152] p. 50
B. Turbulent, vertical wetted wall column Better fit	$N_{Sh,avg} = \dfrac{k'_m d_t}{D} = 0.023 N_{Re}^{0.83} N_{Sc}^{0.44}$ A coefficient 0.0163 has also been reported using $N_{Re'}$, where $v = v$ of gas relative to liquid film. $N_{Sh,avg} = 0.0318 N_{Re}^{0.790} N_{Sc}^{0.5}$	[E] Use with log mean concentration difference for correlations in B and D. N_{Re} is for gas. N_{Sc} for vapor in gas. $2000 < N_{Re} \le 35{,}000$, $0.6 \le N_{Sc} \le 2.5$. Use for gases, d_t = tube diameter. [S] Reevaluated data	[68] [77] p.181 [138] p. 211 [141] p. 265 [149] p. 212 [152] p. 71 [58]
C. Turbulent, very short column	$N_{Sh} = 0.00283 N_{Re,g}^{0.5} N_{Sc,g}^{0.5} N_{Re,liq}^{0.08}$ $N_{Sh} = k_g(d_{tube} - 2\delta)/D$ $N_{Re,g} = \rho_g u_g(d_{tube} - 2\delta)/\mu_g$ $N_{Re,liq} = \rho_{liq} Q_{liq}/[\pi\mu(d_{tube} - 2\delta)]$	[E] Evaporation data $N_{Sh,g}$ = 11 to 65, $N_{Re,g}$ = 2400 to 9100 $N_{Re,liq}$ = 110 to 480, $N_{Sc,g}$ = 0.62 to 1.93 δ = film thickness	[56]
D. Turbulent, vertical wetted wall column with ripples	$N_{Sh,avg} = \dfrac{k'_m d_t}{D} = 0.00814 N_{Re}^{0.83} N_{Sc}^{0.44}\left(\dfrac{4Q\rho}{w\mu}\right)^{0.15}$ $30 \le \left(\dfrac{4Q\rho}{w\mu}\right) < 1200$ $N_{Sh,avg} = \dfrac{k'_m d_t}{D} = 0.023 N_{Re}^{0.8} N_{Sc}^{1/3}$	[E] For gas systems with rippling. Fits 5-18-B for $\left(\dfrac{4Q\rho}{w\mu}\right) = 1000$ [E] "Rounded" approximation to include ripples. Includes solid-liquid mass-transfer data to find ⅓ coefficient on N_{Sc}. May use $N_{Re}^{0.83}$. Use for liquids. See also Table 5-19.	[85] [138] p. 213
E. Turbulent, with ripples	$N_{Sh} = \left(\dfrac{2}{\pi}\right)^{0.5} N_{Re,\dot\varepsilon}^{0.5} N_{Sc}^{0.5}$ $N_{Re,\dot\varepsilon} = \dot\varepsilon L^2/\nu$	[E] $\dot\varepsilon$ = dilation rate of surface = $\dfrac{\partial v_x^c}{\partial x} + \dfrac{\partial v_y^s}{\partial y}$	[150]
F. Rectification in vertical wetted wall column with turbulent vapor flow, Johnstone and Pigford correlation	$N_{Sh,avg} = \dfrac{k'_G d_{col} p_{BM}}{D_v p} = 0.0328(N'_{Re})^{0.77} N_{Sc}^{0.33}$ $3000 < N'_{Re} < 40{,}000$, $0.5 < N_{Sc} < 3$ $N'_{Re} = \dfrac{d_{col} v_{rel} \rho_v}{\mu_v}$, v_{rel} = gas velocity relative to liquid film $= \dfrac{3}{2}\,u_{avg}$ in film	[E] Use logarithmic mean driving force at two ends of column. Based on four systems with gas-side resistance only. p_{BM} = logarithmic mean partial pressure of nondiffusing species B in binary mixture. p = total pressure Modified form is used for structured packings (See Table 5-24-H).	[84] [138] p. 214 [156]

[*]See the beginning of the "Mass Transfer" subsection for references.

TABLE 5-19 Mass-Transfer Correlations for Flow in Pipes and Ducts—Transfer Is from Wall to Fluid

Situation	Correlation	Comments E = Empirical, S = Semiempirical, T = Theoretical	References[a]
A. Tubes, laminar, fully developed parabolic velocity profile, developing concentration profile, constant wall concentration	$N_{Sh} = \dfrac{k'd_t}{D} = 3.66 + \dfrac{0.0668(d_t/x)N_{Re}N_{Sc}}{1 + 0.04[(d_t/x)N_{Re}N_{Sc}]^{2/3}}$	[T] Use log mean concentration difference. For $\dfrac{x/d_t}{N_{Re}N_{Sc}} < 0.10, N_{Re} < 2100.$ x = distance from tube entrance. Good agreement with experiment at values $10^4 > \dfrac{\pi}{4}\dfrac{d_t}{x}N_{Re}N_{Sc} > 10$	[77] p. 176 [87] p. 525 [141] p. 159
Fully developed concentration profile	$N_{Sh} = \dfrac{k'd_t}{D} = 3.66$	[T] $\dfrac{x/d_t}{N_{Re}N_{Sc}} > 0.1$	[141] p. 165
B. Tubes, approximate solution	$N_{Sh,x} = \dfrac{k'd_t}{D} = 1.077\left(\dfrac{d_t}{x}\right)^{1/3}(N_{Re}N_{Sc})^{1/3}$ $N_{Sh,avg} = \dfrac{k'd_t}{D} = 1.615\left(\dfrac{d_t}{L}\right)^{1/3}(N_{Re}N_{Sc})^{1/3}$	[T] For arithmetic concentration difference. $\dfrac{W}{\rho Dx} > 400$ Leveque's approximation: Concentration BL is thin. Assume velocity profile is linear. High mass velocity. Fits liquid data well.	[141] p. 166
C. Tubes, laminar, uniform plug velocity, developing concentration profile, constant wall concentration	$N_{Sh,avg} = \dfrac{1}{2}\dfrac{d_t}{L}N_{Re}N_{Sc}\left[\dfrac{1 - 4\sum_{j=1}^{\infty}a_j^{-2}\exp\left(\dfrac{-2a_j^2(x/r_t)}{N_{Re}N_{Sc}}\right)}{1 + 4\sum_{j=1}^{\infty}a_j^{-2}\exp\left(\dfrac{-2a_j^2(x/r_t)}{N_{Re}N_{Sc}}\right)}\right]$ Graetz solution for heat transfer written for M.T.	[T] Use arithmetic concentration difference. Fits gas data well, for $\dfrac{W}{D\rho x} < 50$ (fit is fortuitous). $N_{Sh,avg} = (k'_m d_t)/D.$ $a_1 = 2.405, a_2 = 5.520,$ $a_3 = 8.654, a_4 = 11.792, a_5 = 14.931.$ Graphical solutions are in references.	[103] [141] p. 150
D. Laminar, fully developed parabolic velocity profile, constant mass flux at wall	$N_{Sh,x} = \left[\dfrac{11}{48} - \dfrac{1}{2}\sum_{j=1}^{\infty}\dfrac{\exp[-\lambda_j^2(x/r_t)/(N_{Re}N_{Sc})]}{C_j\lambda_j^4}\right]^{-1}$ $\begin{array}{ccc} j & \lambda_j^2 & c_j \\ 1 & 25.68 & 7.630 \times 10^{-3} \\ 2 & 83.86 & 2.058 \times 10^{-3} \\ 3 & 174.2 & 0.901 \times 10^{-3} \\ 4 & 296.5 & 0.487 \times 10^{-3} \\ 5 & 450.9 & 0.297 \times 10^{-3} \end{array}$	[T] Use log mean concentration difference. $N_{Re} < 2100$ $N_{Sh,x} = \dfrac{k'd_t}{D}$ $N_{Re} = \dfrac{vd_t\rho}{\mu}$	[139] [141] p. 167
E. Laminar, alternate	$N_{Sh} = 4.36 + \dfrac{0.023(d_t/L)N_{Re}N_{Sc}}{1 + 0.0012(d_t/L)N_{Re}N_{Sc}}$	[T] $N_{sh} = \dfrac{k'd_t}{D},$ Use log mean concentration difference. $N_{Re} < 2100$	[77] p. 176
F. Laminar, fully developed concentration and velocity profile	$N_{Sh} = \dfrac{k'd_t}{D} = \dfrac{48}{11} = 4.3636$	[T] Use log mean concentration difference. $N_{Re} < 2100$	[141] p. 167
G. Vertical tubes, laminar flow, forced and natural convection	$N_{Sh,avg} = 1.62N_{Gz}^{1/3}\left[1 \pm 0.0742\dfrac{(N_{Gr}N_{Sc}d/L)^{3/4}}{N_{Gz}}\right]^{1/3}$	[T] Approximate solution. Use minus sign if forced and natural convection oppose each other. Good agreement with experiment. $N_{Gz} = \dfrac{N_{Re}N_{Sc}d}{L}, N_{Gr} = \dfrac{g\Delta\rho d^3}{\rho v^2}$	[127]
H. Hollow-fiber extraction inside fibers	$N_{Sh} = 0.5N_{Gz}, N_{Gz} < 6$ $N_{Sh} = 1.62N_{Gz}^{0.5}, N_{Gz} \geq 6$	[E] Use arithmetic concentration difference.	[41]
I. Tubes, laminar, RO systems	$N_{Sh,avg} = \dfrac{k'_m d_t}{D} = 1.632\left(\dfrac{ud_t^2}{DL}\right)^{1/3}$	Use arithmetic concentration difference. Thin concentration polarization layer, not fully developed. $N_{Re} < 2000, L$ = length tube.	[40]
J. Tubes and parallel plates, laminar RO	Graphical solutions for concentration polarization. Uniform velocity through walls.	[T]	[137]
K. Rotating annulus for reverse osmosis	For nonvortical flow: $N_{Sh} = 2.15\left[N_{Ta}\left(\dfrac{d}{r_i}\right)^{0.5}\right]^{0.18}N_{Sc}^{1/3}$ For vortical flow: $N_{Sh} = 1.05\left[N_{Ta}\left(\dfrac{d}{r_i}\right)^{0.5}\right]N_{Sc}^{1/3}$	[E,S] N_{Ta} = Taylor number = $r_i\omega d/\nu$ r_i = inner cylinder radius ω = rotational speed, rad/s d = gap width between cylinders	[100]

TABLE 5-19 Mass-Transfer Correlations for Flow in Pipes and Ducts—Transfer Is from Wall to Fluid (*Continued*)

Situation	Correlation	Comments E = Empirical, S = Semiempirical, T = Theoretical	References*
L. Parallel plates, laminar, parabolic velocity, developing concentration profile, constant wall concentration	Graphical solution	[T] Low transfer rates.	[141] p. 176
L′. 5-19-L, fully developed	$N_{Sh} = \dfrac{k'(2h)}{D} = 7.6$	[T] h = distance between plates. Use log mean concentration difference. $\dfrac{N_{Re}N_{Sc}}{x/(2h)} < 20$	[141] p. 177
M. Parallel plates, laminar, parabolic velocity, developing concentration profile, constant mass flux at wall	Graphical solution	[T] Low transfer rates.	[141] p. 176
N. 5-19-M, fully developed	$N_{Sh} = \dfrac{k'(2h)}{D} = 8.23$	[T] Use log mean concentration difference. $\dfrac{N_{Re}N_{Sc}}{x/(2h)} < 20$	[141] p. 177
O. Laminar flow, vertical parallel plates, forced and natural convection	$N_{Sh,avg} = 1.47 N_{Gz}^{1/3}\left[1 \pm 0.0989 \dfrac{(N_{Gr}N_{Sc}\, h/L)^{3/4}}{N_{Gz}} \right]^{1/3}$	[T] Approximate solution. Use minus sign if forced and natural convection oppose each other. Good agreement with experiment. $N_{Gz} = \dfrac{N_{Re}N_{Sc}h}{L},\ N_{Gr} = \dfrac{g\Delta\rho h^3}{\rho\nu^2}$	[127]
P. Parallel plates, laminar, RO systems	$N_{Sh,avg} = \dfrac{k'(2H_p)}{D} = 2.354\left(\dfrac{uH_p^2}{DL}\right)^{1/3}$	Thin concentration polarization layer. Short tubes, concentration profile not fully developed. Use arithmetic concentration difference.	[40]
Q. Tubes, turbulent	$N_{Sh,avg} = \dfrac{k'_m d_t}{D} = 0.023 N_{Re}^{0.83} N_{Sc}^{1/3}$ $2100 < N_{Re} < 35{,}000$ $0.6 < N_{Sc} < 3000$ $N_{Sh,avg} = \dfrac{k'_m d_t}{D} = 0.023 N_{Re}^{0.83} N_{Sc}^{0.44}$ $2000 < N_{Re} < 35{,}000$ $0.6 < N_{Sc} < 2.5$	[E] Use with log mean concentration difference at two ends of tube. Good fit for liquids. From wetted wall column and dissolution data— see Table 5-18-B. [E] Evaporation of liquids. Use with log mean concentration difference. Better fit for gases.	[77] p. 181 [103] [152] p. 72 [68][77] p. 181 [88] p. 112 [138] p. 211
R. Tubes, turbulent	$N_{Sh} = \dfrac{k'd_t}{D} = 0.0096 N_{Re}^{0.913} N_{Sc}^{0.346}$	[E] $430 < N_{Sc} < 100{,}000$. Dissolution data. Use for high N_{Sc}.	[105] p. 668
S. Tubes, turbulent, smooth tubes, Reynolds analogy	$N_{Sh} = \dfrac{k'd_t}{D} = \left(\dfrac{f}{2}\right) N_{Re}N_{Sc}$ f = Fanning friction faction	[T] Use arithmetic concentration difference. N_{Sc} near 1.0 Turbulent core extends to wall. Of limited utility.	[66] p. 474 [77] p. 171 [141] p. 239 [149] p. 250
T. Tubes, turbulent, smooth tubes, Chilton-Colburn analogy	$j_D = j_H \leqslant \dfrac{f}{2}$ If $\dfrac{f}{2} = 0.023 N_{Re}^{-0.2}$, $j_D = \dfrac{N_{Sh}}{N_{Re}N_{Sc}^{1/3}} = 0.023 N_{Re}^{-0.2}$ $N_{Sh} = \dfrac{k'd_t}{D}$, Sec. 5-17-G $j_D = j_H = f(N_{Re},\ \text{geometry and B.C.})$	[E] Use log-mean concentration difference. Relating j_D to $f/2$ approximate. N_{Pr} and N_{Sc} near 1.0. Low concentration. Results about 20% lower than experiment. $3 \times 10^4 < N_{Re} < 10^6$ [E] Good over wide ranges.	[39] pp. 400, 647 [51][53] [141] p. 264 [149] p. 251 [66] p. 475 [39] p. 647 [51]
U. Tubes, turbulent, smooth tubes, constant surface concentration, Prandtl analogy	$N_{Sh} = \dfrac{k'd_t}{D} = \dfrac{(f/2)N_{Re}N_{Sc}}{1 + 5\sqrt{f/2}(N_{Sc} - 1)}$ $\dfrac{f}{2} = 0.04 N_{Re}^{-0.25}$	[T] Use arithmetic concentration difference. Improvement over Reynolds analogy. Best for N_{Sc} near 1.0.	[77] p. 173 [141] p. 241

TABLE 5-19   Mass-Transfer Correlations for Flow in Pipes and Ducts—Transfer Is from Wall to Fluid (*Concluded*)

Situation	Correlation	Comments E = Empirical, S = Semiempirical, T = Theoretical	References[*]
V. Tubes, turbulent, smooth tubes, Constant surface concentration, Von Karman analogy	$$N_{Sh} = \frac{(f/2)N_{Re}N_{Sc}}{1 + 5\sqrt{f/2}\left\{(N_{Sc} - 1) + \ln\left[1 + \frac{5}{6}(N_{Sc} - 1)\right]\right\}}$$ $$\frac{f}{2} = 0.04 N_{Re}^{-0.25}$$	[T] Use arithmetic concentration difference. $N_{Sh} = k'd_t/D$. Improvement over Prandtl, $N_{Sc} < 25$.	[77] p. 173 [141] p. 243 [149] p. 250 [154]
W. Tubes, turbulent, smooth tubes, constant surface concentration	For $0.5 < N_{Sc} < 10$: $N_{Sh,avg} = 0.0097 N_{Re}^{9/10} N_{Sc}^{1/2}$ $\times (1.10 + 0.44 N_{Sc}^{-1/3} - 0.70 N_{Sc}^{-1/6})$ For $10 < N_{Sc} < 1000$: $N_{Sh,avg}$ $= \dfrac{0.0097 N_{Re}^{9/10} N_{Sc}^{1/2}(1.10 + 0.44 N_{Sc}^{-1/3} - 0.70 N_{Sc}^{-1/6})}{1 + 0.064 N_{Sc}^{1/2}(1.10 + 0.44 N_{Sc}^{-1/3} - 0.70 N_{Sc}^{-1/6})}$ For $N_{Sc} > 1000$: $N_{Sh,avg} = 0.0102 N_{Re}^{9/10} N_{Sc}^{1/3}$	[S] Use arithmetic concentration difference. Based on partial fluid renewal and an infrequently replenished thin fluid layer for high N_{Sc}. Good fit to available data. $N_{Re} = \dfrac{u_{bulk}d_t}{\nu}$ $N_{Sh,avg} = \dfrac{k'_{avg}d_t}{D}$	[77] p. 179 [117]
X. Turbulent flow, tubes	$$N_{St} = \frac{N_{Sh}}{N_{Pe}} = \frac{N_{Sh}}{N_{Re}N_{Sc}} = 0.0149 N_{Re}^{-0.12} N_{Sc}^{-2/3}$$	[E] Smooth pipe data. Data fits within 4% except at $N_{Sc} > 20{,}000$, where experimental data is underpredicted. $N_{Sc} > 100$, $10^5 > N_{Re} > 2100$	[107]
Y. Turbulent flow, noncircular ducts	Use correlations with $$d_{eq} = \frac{4\ \text{cross-sectional area}}{\text{wetted perimeter}}$$	Can be suspect for systems with sharp corners. Parallel plates: $$d_{eq} = 4\ \frac{2hw}{2w + 2h}$$	[141] p. 289
Z. Decaying swirling flow in pipe	$N_{Sh,avg} = 0.3508 N_{Sc}^{1/3} N_{Re}^{0.759}(x/d)^{-0.400} \times (1 + \tan\theta)^{0.271}$ $N_{Re} = 1730$ to 8650, $N_{Sc} = 1692$	[E,S] x = axial distance, d = diameter, θ = vane angle (15° to 60°) Regression coefficient = 0.9793. Swirling increases mass transfer.	[161]

[*]See the beginning of the "Mass Transfer" subsection for references.

TABLE 5-20   Mass-Transfer Correlations for Flow Past Submerged Objects

Situation	Correlation	Comments E = Empirical, S = Semiempirical, T = Theoretical	References°
A. Single sphere	$$N_{Sh} = \frac{k'_G p_{BLM} RT d_s}{PD} = \frac{2r}{r - r_s}$$ $\begin{array}{c\|c\|c\|c\|c\|c} r/r_s & 2 & 5 & 10 & 50 & \infty \text{ (asymptotic limit)} \\ \hline N_{Sh} & 4.0 & 2.5 & 2.22 & 2.04 & 2.0 \end{array}$	[T] Use with log mean concentration difference. r = distance from sphere, r_s, d_s = radius and diameter of sphere. No convection.	[141] p. 18
B. Single sphere, creeping flow with forced convection	$$N_{Sh} = \frac{k'd}{D} = [4.0 + 1.21(N_{Re}N_{Sc})^{2/3}]^{1/2}$$ $$N_{Sh} = \frac{k'd}{D} = a(N_{Re}N_{Sc})^{1/3}$$ $a = 1.00 \pm 0.01$	[T] Use with log mean concentration difference. Average over sphere. Numerical calculations. $(N_{Re}N_{Sc}) < 10{,}000 \ N_{Re} < 1.0$. Constant sphere diameter. Low mass-transfer rates. [T] Fit to above ignoring molecular diffusion. $1000 < (N_{Re}N_{Sc}) < 10{,}000$.	[46][88] p. 114 [105] [138] p. 214 [101] p. 80 [138] p. 215
C. Single spheres, molecular diffusion, and forced convection, low flow rates	$N_{Sh} = 2.0 + AN_{Re}^{1/2}N_{Sc}^{1/3}$ $A = 0.5$ to 0.62 $A = 0.60$. $A = 0.95$. $A = 0.95$. $A = 0.544$.	[E] Use with log mean concentration difference. Average over sphere. Frössling Eq. ($A = 0.552$), $2 \le N_{Re} \le 800$, $0.6 \le N_{Sc} \le 2.7$. N_{Sh} lower than experimental at high N_{Re}. [E] Ranz and Marshall $2 \le N_{Re} \le 200$, $0.6 \le N_{Sc} \le 2.5$. Modifications recommended [110] See also Table 5-23-O. [E] Liquids $2 \le N_{Re} \le 2{,}000$. Graph in Ref. 138, p. 217–218. [E] $100 \le N_{Re} \le 700$; $1{,}200 \le N_{Sc} \le 1525$. [E] Use with arithmetic concentration difference. $N_{Sc} = 1$; $50 \le N_{Re} \le 350$.	[39] [77], p. 194 [88] p. 114 [141] p. 276 [39] p. 409, 647 [121] [110] [138] p. 217 [141] p. 276 [65][66] p. 482 [138] p. 217 [126][141] p. 276 [81][141] p. 276
D. Same as 5-20-C	$$N_{Sh} = \frac{k'd_s}{D} = 2.0 + 0.575N_{Re}^{1/2}N_{Sc}^{0.35}$$	[E] Use with log mean concentration difference. $N_{Sc} \le 1$, $N_{Re} < 1$.	[70][141] p. 276
E. Same as 5-20-C	$$N_{Sh} = \frac{k'd_s}{D} = 2.0 + 0.552N_{Re}^{0.53}N_{Sc}^{1/3}$$	[E] Use with log mean concentration difference. $1.0 < N_{Re} < 48{,}000$ Gases: $0.6 \le N_{Sc} \le 2.7$.	[66] p. 482
F. Single spheres, forced concentration, any flow rate	$$N_{Sh} = \frac{k'_L d_s}{D} = 2.0 + 0.59\left[\frac{E^{1/3}d_p^{4/3}\rho}{\mu}\right]^{0.57}N_{Sc}^{1/3}$$ Energy dissipation rate per unit mass of fluid (ranges $570 < N_{Sc} < 1420$): $$E = \left(\frac{C_{Dr}}{2}\right)\left(\frac{v_r^3}{d_p}\right)\frac{m^2}{s^3}$$	[S] Correlates large amount of data and compares to published data. v_r = relative velocity between fluid and sphere, m/s. C_{Dr} = drag coefficient for single particle fixed in fluid at velocity v_r. See 5-23-F for calculation details and applications. $2 < \left(\dfrac{E^{1/3}d_p^{4/3}\rho}{\mu}\right) < 63{,}000$	[108]
G. Single spheres, forced convection, high flow rates, ignoring molecular diffusion	$$N_{Sh} = \frac{k'd_s}{D} = 0.347N_{Re}^{0.62}N_{Sc}^{1/3}$$ $$N_{Sh} = \frac{k'd_s}{D} = 0.33N_{Re}^{0.6}N_{Sc}^{1/3}$$ $$N_{Sh} = \frac{k'd_s}{D} = 0.43N_{Re}^{0.56}N_{Sc}^{1/3}$$ $$N_{Sh} = \frac{k'd_s}{D} = 0.692N_{Re}^{0.514}N_{Sc}^{1/3}$$	[E] Use with arithmetic concentration difference. Liquids, $2000 < N_{Re} < 17{,}000$. High N_{Sc}, graph in Ref. 138, p. 217–218. [E] $1500 \le N_{Re} \le 12{,}000$. [E] $200 \le N_{Re} \le 4 \times 10^4$, "air" $\le N_{Sc} \le$ "water." [E] $500 \le N_{Re} \le 5000$.	[66] p. 482 [147] [138] p. 217 [141] p. 276 [141] p. 276 [112] [141] p. 276
H. Single sphere immersed in bed of smaller particles. For gases.	$$N_{Sh,avg} = \frac{kd_1}{D'} = \varepsilon\left[4 + \frac{4}{5}N_{Pe'}^{2/3} + \frac{4}{\pi}N_{Pe'}\right]$$ $$\left(1 + \frac{1}{9}N_{Pe'}\right)^{1/2}$$ Limit $N_{Pe'} \to 0, N_{Sh,avg} = 2\varepsilon$	[T] Compared to experiment. $N_{Pe'} = \dfrac{u_o d_1}{D'}$, $D' = D/\tau$, D = molecular diffusivity, d_1 = diameter large particle, τ = tortuosity. Arithmetic conc. difference fluid flow in inert bed follows Darcy's law.	[71]
I. Single cylinders, perpendicular flow	$$N_{Sh} = \frac{k'd_s}{D} = AN_{Re}^{1/2}N_{Sc}^{1/3}, A = 0.82$$ $A = 0.74$ $A = 0.582$ $j_D = 0.600(N_{Re})^{-0.487}$ $$N_{Sh} = \frac{k'd_{cyl}}{D}$$	[E] $100 < N_{Re} \le 3500$, $N_{Sc} = 1560$. [E] $120 \le N_{Re} \le 6000$, $N_{Sc} = 2.44$. [E] $300 \le N_{Re} \le 7600$, $N_{Sc} = 1200$. [E] Use with arithmetic concentration difference. $50 \le N_{Re} \le 50{,}000$; gases, $0.6 \le N_{Sc} \le 2.6$; liquids; $1000 \le N_{Sc} \le 3000$. Data scatter $\pm 30\%$.	[141] p. 276 [141] p. 276 [142] [141] p. 276 [66] p. 486

TABLE 5-20   Mass-Transfer Correlations for Flow Past Submerged Objects (*Concluded*)

Situation	Correlation	Comments E = Empirical, S = Semiempirical, T = Theoretical	References[*]
J. Rotating cylinder in an infinite liquid, no forced flow	$j_D' = \dfrac{k'}{v}\,N_{Sc}^{0.644} = 0.0791 N_{Re}^{-0.30}$ Results presented graphically to $N_{Re} = 241{,}000$. $N_{Re} = \dfrac{vd_{cyl}\mu}{\rho}$ where $v = \dfrac{\omega d_{cyl}}{2}$ = peripheral velocity	[E] Used with arithmetic concentration difference. Useful geometry in electrochemical studies. $112 < N_{Re} \le 100{,}000.\ 835 < N_{Sc} < 11490$ k' = mass-transfer coefficient, cm/s; ω = rotational speed, radian/s.	[60] [138] p. 238
K. Stationary or rotating cylinder for air	Stationary: $N_{Sh,avg} = AN_{Re}^{c}S_c^{1/3}$ $2.0 \times 10^4 \le N_{Re} \le 2.5 \times 10^5; d/H = 0.3, \text{Tu} = 0.6\%$ $A = 0.0539, c = 0.771$ [114] A and c depend on geometry [37] Rotating in still air: $N_{Sh,avg} = 0.169 N_{Re,\omega}^{2/3}$ $1.0\text{E4} \le N_{Re,\omega} \le 1.0\text{E5}; N_{Sc} \approx 2.0; N_{Gr} \approx 2.0 \times 10^6$	[E] Reasonable agreement with data of other investigators. d = diameter of cylinder, H = height of wind tunnel, Tu of = turbulence level, N_{Re}w = rotational Reynold's number = $u_\omega d\rho/\mu$, u_ω = cylinder surface velocity. Also correlations for two-dimensional slot jet flow [114]. For references to other correlations see [37].	[37] [114]
L. Oblate spheroid, forced convection	$j_D = \dfrac{N_{Sh}}{N_{Re}N_{Sc}^{1/3}} = 0.74 N_{Re}^{-0.5}$ $N_{Re} = \dfrac{d_{ch}\,v\rho}{\mu},\ d_{ch} = \dfrac{\text{total surface area}}{\text{perimeter normal to flow}}$ e.g., for cube with side length a, $d_{ch} = 1.27a$. $N_{Sh} = \dfrac{k'd_{ch}}{D}$	[E] Used with arithmetic concentration difference. $120 \le N_{Re} \le 6000$; standard deviation 2.1%. Eccentricities between 1:1 (spheres) and 3:1. Oblate spheroid is often approximated by drops.	[141] p. 284 [142]
M. Other objects, including prisms, cubes, hemispheres, spheres, and cylinders; forced convection	$j_D = 0.692 N_{Re,p}^{-0.486},\ N_{Re,p} = \dfrac{vd_{ch}\rho}{\mu}$ Terms same as in 5-20-J.	[E] Used with arithmetic concentration difference. Agrees with cylinder and oblate spheroid results, $\pm 15\%$. Assumes molecular diffusion and natural convection are negligible. $500 \le N_{Re,p} \le 5000$. Turbulent.	[88] p. 115 [141] p. 285 [111] [112]
N. Other objects, molecular diffusion limits	$N_{Sh} = \dfrac{k'd_{ch}}{D} = A$	[T] Use with arithmetic concentration difference. Hard to reach limits in experiments. Spheres and cubes $A = 2$, tetrahedrons $A = 2\sqrt{6}$ octahedrons $2\sqrt{2}$.	[88] p. 114
O. Shell side of microporous hollow fiber module for solvent extraction	$N_{Sh} = \beta[d_h(1-\varphi)/L]N_{Re}^{0.6}N_{Sc}^{0.33}$ $N_{Sh} = \dfrac{\overline{K}d_h}{D}$ $N_{Re} = \dfrac{d_h v\rho}{\mu},\ \overline{K}$ = overall mass-transfer coefficient $\beta = 5.8$ for hydrophobic membrane. $\beta = 6.1$ for hydrophilic membrane.	[E] Use with logarithmic mean concentration difference. d_h = hydraulic diameter $= \dfrac{4 \times \text{cross-sectional area of flow}}{\text{wetted perimeter}}$ φ = packing fraction of shell side. L = module length. Based on area of contact according to inside or outside diameter of tubes depending on location of interface between aqueous and organic phases. Can also be applied to gas-liquid systems with liquid on shell side.	[118]

See Table 5-23 for flow in packed beds.
[*]See the beginning of the "Mass Transfer" subsection for references.

TABLE 5-21 Mass-Transfer Correlations for Drops, Bubbles, and Bubble Columns

Conditions	Correlations	Comments E = Empirical, S = Semiempirical, T = Theoretical	References[a]
A. Single liquid drop in immiscible liquid, drop formation, discontinuous (drop) phase coefficient	$\hat{k}_{d,f} = A\left(\dfrac{\rho_d}{M_d}\right)_{av}\left(\dfrac{D_d}{\pi t_f}\right)^{1/2}$ $A = \dfrac{24}{7}$ (penetration theory) $A = 1.31$ (semiempirical value) $A = \left[\dfrac{24}{7}(0.8624)\right]$ (extension by fresh surface elements)	[T,S] Use arithmetic mole fraction difference. Fits some, but not all, data. Low mass transfer rate. M_d = mean molecular weight of dispersed phase; t_f = formation time of drop. $k_{L,d}$ = mean dispersed liquid phase M.T. coefficient kmole/[s·m² (mole fraction)].	[141] p. 399
B. Same as 5-21-A	$\hat{k}_{d,f} = 0.0432$ $\times \dfrac{d_p}{t_f}\left(\dfrac{\rho_d}{M_d}\right)_{av}\left(\dfrac{u_o}{d_pg}\right)^{0.089}\left(\dfrac{d_p^2}{t_fD_d}\right)^{-0.334}\left(\dfrac{\mu_d}{\sqrt{\rho_d d_p \sigma g_c}}\right)^{-0.601}$	[E] Use arithmetic mole fraction difference. Based on 23 data points for 3 systems. Average absolute deviation 26%. Use with surface area of drop after detachment occurs. u_o = velocity through nozzle; σ = interfacial tension.	[141] p. 401 [144] p. 434
C. Single liquid drop in immiscible liquid, drop formation, continuous phase coefficient	$\hat{k}_{c,f} = 4.6\left(\dfrac{\rho_c}{M_c}\right)_{av}\sqrt{\dfrac{D_c}{\pi t_f}}$	[T] Use arithmetic mole fraction difference. Based on rate of bubble growth away from fixed orifice. Approximately three times too high compared to experiments.	[141] p. 402
D. Same as 5-21-C	$k_{L,c} = 0.386$ $\times \left(\dfrac{\rho_c}{M_c}\right)_{av}\left(\dfrac{D_c}{t_f}\right)^{0.5}\left(\dfrac{\rho_c\sigma g_c}{\Delta\rho gt_f\mu_c}\right)^{0.407}\left(\dfrac{gt_f^2}{d_p}\right)^{0.148}$	[E] Average absolute deviation 11% for 20 data points for 3 systems.	[141] p. 402 [144] p. 434
E. Single liquid drop in immiscible liquid, free rise or fall, discontinuous phase coefficient, stagnant drops	$k_{L,d,m} = \dfrac{-d_p}{6t}\left(\dfrac{\rho_d}{M_d}\right)_{av}\ln\left\{\dfrac{6}{\pi^2}\sum_{j=1}^{\infty}\dfrac{1}{j^2}\exp\left[\left(\dfrac{-D_dj^2\pi^2t}{(d_p/2)^2}\right)\right]\right\}$	[T] Use with log mean mole fraction differences based on ends of column. t = rise time. No continuous phase resistance. Stagnant drops are likely if drop is very viscous, quite small, or is coated with surface active agent. $k_{L,d,m}$ = mean dispersed liquid M.T. coefficient.	[141] p. 404 [144] p. 435
F. Same as 5-21-E	$\hat{k}_{L,d,m} = \dfrac{-d_p}{6t}\left(\dfrac{\rho_d}{M_d}\right)_{av}\ln\left[1 - \dfrac{\pi D_d^{1/2}t^{1/2}}{d_p/2}\right]$	[S] See 5-21-E. Approximation for fractional extractions less than 50%.	[141] p. 404 [144] p. 435
G. Same as 5-21-E, continuous phase coefficient, stagnant drops, spherical	$N_{Sh} = \dfrac{k_{L,c,m}d_c}{D_c} = 0.74\left(\dfrac{\rho_c}{M_c}\right)_{av}N_{Re}^{1/2}(N_{Sc})^{1/3}$	[E] $N_{Re} = \dfrac{v_s d_p\rho_c}{\mu_c}$, v_s = slip velocity between drop and continuous phase.	[141] p. 407 [142][144] p. 436
H. Single bubble or drop with surfactant. Stokes flow.	$N_{Sh} = 2.0 + \alpha N_{Pe}^\beta$, $N_{Sh} = 2rk/D$ $\alpha = \dfrac{5.49}{A + 6.10} + \dfrac{A}{A + 28.64}$ $\beta = \dfrac{0.35A + 17.21}{A + 34.14}$ $2r = 2$ to $50\ \mu m$, $A = 2.8E4$ to $7.0E5$ $0.0026 < N_{Pe,s} < 340$, $2.1 < N_{Ma} < 1.3E6$ $N_{Pe} = 1.0$ to 2.5×10^4, $N_{Re} = 2.2 \times 10^{-6}$ to 0.034	[T] A = surface retardation parameter $A = B\Gamma_o r/\mu D_s = N_{Ma}N_{Pe,s}$ $N_{Ma} = B\Gamma_o/\mu u$ = Marangoni no. Γ = surfactant surface conc. $N_{Pe,s}$ = surface Peclet number = ur/D_s D_s = surface diffusivity N_{Pe} = bulk Peclet number For $A \gg 1$ acts like rigid sphere: $\beta \to 0.35$, $\alpha \to 1/2864 = 0.035$	[120]
I. 5-21-E, oblate spheroid	$N_{Sh} = \dfrac{k_{L,c,m}d_3}{D_c} = 0.74\left(\dfrac{\rho_c}{M_c}\right)_{av}(N_{Re,3})^{1/2}(N_{Sc,c})^{1/3}$ $N_{Re,3} = \dfrac{v_s d_3\rho_c}{\mu_c}$	[E] Used with log mean mole fraction. Differences based on ends of extraction column; 100 measured values ±2% deviation. Based on area oblate spheroid. v_s = slip velocity, $d_3 = \dfrac{\text{total drop surface area}}{\text{perimeter normal to flow}}$	[141] p. 285, 406, 407
J. Single liquid drop in immiscible liquid, Free rise or fall, discontinuous phase coefficient, circulating drops	$k_{dr,circ} = -\dfrac{d_p}{6\theta}\ln\left[\dfrac{3}{8}\sum_{j=1}^{\infty}B_j^2\exp\left(-\dfrac{\lambda_j 64D_d\theta}{d_p^2}\right)\right]$ Eigenvalues for Circulating Drop 	$k_d d_p/D_d$	λ_1	λ_2	λ_3	B_1	B_2	B_3
---	---	---	---	---	---	---
3.20	0.262	0.424		1.49	0.107	
10.7	0.680	4.92		1.49	0.300	
26.7	1.082	5.90	15.7	1.49	0.495	0.205
107	1.484	7.88	19.5	1.39	0.603	0.384
320	1.60	8.62	21.3	1.31	0.583	0.391
∞	1.656	9.08	22.2	1.29	0.596	0.386		[T] Use with arithmetic concentration difference. θ = drop residence time. A more complete listing of eigenvalues is given by Refs. 62 and 76. $k'_{L,d,circ}$ is m/s.	[62][76][141] p. 405 [152] p. 523

TABLE 5-21 **Mass-Transfer Correlations for Drops, Bubbles, and Bubble Columns** (*Continued*)

Conditions	Correlations	Comments E = Empirical, S = Semiempirical, T = Theoretical	References*
K. Same as 5-21-J	$\hat{k}_{L,d,\text{circ}} = -\dfrac{d_p}{6\theta}\left(\dfrac{\rho_d}{M_d}\right)_{av}\ln\left[1 - \dfrac{R^{1/2}\pi D_d^{1/2}\theta^{1/2}}{d_p/2}\right]$	[E] Used with mole fractions for extraction less than 50%, $R \approx 2.25$.	[141] p. 405
L. Same as 5-21-J	$N_{Sh} = \dfrac{\hat{k}_{L,d,\text{circ}}d_p}{D_d}$ $= 31.4\left(\dfrac{\rho_d}{M_f}\right)_{av}\left(\dfrac{4D_d t}{d_p^2}\right)^{-0.34} N_{Sc,d}^{-0.125}\left(\dfrac{d_p v_s^2 \rho_c}{\sigma g_c}\right)^{-0.37}$	[E] Used with log mean mole fraction difference. d_p = diameter of sphere with same volume as drop. $856 \le N_{Sc} \le 79{,}800$, $2.34 \le \sigma \le 4.8$ dynes/cm.	[144] p. 435 [145]
M. Liquid drop in immiscible liquid, free rise or fall, continuous phase coefficient, circulating single drops	$N_{Sh,c} = \dfrac{k'_{L,c}d_p}{D_d}$ $= \left[2 + 0.463 N_{Re,\text{drop}}^{0.484} N_{Sc,c}^{0.339}\left(\dfrac{d_p g^{1/3}}{D_c^{2/3}}\right)^{0.072}\right]F$ $F = 0.281 + 1.615K + 3.73K^2 - 1.874K$ $K = N_{Re,\text{drop}}^{1/8}\left(\dfrac{\mu_c}{\mu_d}\right)^{1/4}\left(\dfrac{\mu_c v_s}{\sigma g_c}\right)^{1/6}$	[E] Used as an arithmetic concentration difference. $N_{Re,\text{drop}} = \dfrac{d_p v_s \rho_c}{\mu_c}$ Solid sphere form with correction factor F.	[82]
N. Same as 5-21-M, circulating, single drop	$N_{Sh} = \dfrac{k_{L,c}d_p}{D_c} = 0.6\left(\dfrac{\rho_c}{M_c}\right)_{av}N_{Re,\text{drop}}^{1/2}N_{Sc,c}^{1/2}$	[E] Used as an arithmetic concentration difference. Low σ.	[141] p. 407
O. Same as 5-21-M, circulating swarm of drops	$k_{L,c} = 0.725\left(\dfrac{\rho_c}{M_c}\right)_{av}N_{Re,\text{drop}}^{-0.43}N_{Sc,c}^{-0.58}v_s(1-\phi_d)$	[E] Used as an arithmetic concentration difference. Low σ, disperse-phase holdup of drop swarm. ϕ_d = volume fraction dispersed phase.	[141] p. 407 [144] p. 436
P. Liquid drops in immiscible liquid, free rise or fall, discontinuous phase coefficient, oscillating drops	$N_{Sh} = \dfrac{k_{L,d,\text{osc}}d_p}{D_d}$ $= 0.32\left(\dfrac{\rho_d}{M_d}\right)_{av}\left(\dfrac{4D_d t}{d_p^2}\right)^{-0.14}N_{Re,\text{drop}}^{0.68}\left(\dfrac{\sigma^3 g_c^3 \rho_c^2}{g\mu_c^4\Delta\rho}\right)^{0.10}$	[E] Used with a log mean mole fraction difference. Based on ends of extraction column. $N_{Re,\text{drop}} = \dfrac{d_p v_s \rho_c}{\mu_c}$, $411 \le N_{Re} \le 3114$ d_p = diameter of sphere with volume of drop. Average absolute deviation from data, 10.5%. Low interfacial tension (3.5–5.8 dyn), $\mu_c < 1.35$ centipoise.	[141] p. 406 [144] p. 435 [145]
Q. Same as 5-21-P	$k_{L,d,\text{osc}} = \dfrac{0.00375 v_s}{1 + \mu_d/\mu_c}$	[T] Use with log mean concentration difference. Based on end of extraction column. No continuous phase resistance. $k_{L,d,\text{osc}}$ in cm/s, v_s = drop velocity relative to continuous phase.	[138] p. 228 [141] p. 405
R. Single liquid drop in immiscible liquid, range rigid to fully circulating	$N_{Sh,c,\text{rigid}} = \dfrac{k_c d_p}{D_c} = 2.43 + 0.774 N_{Re}^{0.5}N_{Sc}^{0.33}$ $\qquad\qquad + 0.0103 N_{Re}N_{Sc}^{0.33}$ $N_{Sh,c,\text{fully circular}} = \left[\dfrac{2}{\pi^{0.5}}\right]N_{Pe,c}^{0.5}$ Drops in intermediate range: $\dfrac{N_{Sh,c} - N_{Sh,c,\text{rigid}}}{N_{Sh,c,\text{fully circular}} - N_{Sh,c,\text{rigid}}} = 1 - \exp\left[-(4.18\times 10^{-3})N_{Pe,c}^{0.42}\right]$	[E] Allows for slight effect of wake. Rigid drops: $10^4 < N_{Pe,c} < 10^6$ Circulating drops: $10 < N_{Re} < 1200$, $190 < N_{Sc} < 241{,}000$, $10^3 < N_{Pe,c} < 10^6$	[146] p. 58
S. Coalescing drops in immiscible liquid, discontinuous phase coefficient	$\hat{k}_{d,\text{coal}} = 0.173\dfrac{d_p}{t_f}\left(\dfrac{\rho_d}{M_d}\right)_{av}\left(\dfrac{\mu_d}{\rho_d D_d}\right)^{-1.115}$ $\qquad \times \left(\dfrac{\Delta\rho g d_p^2}{\sigma g_c}\right)^{1.302}\left(\dfrac{v_s^2 t_f}{D_d}\right)^{0.146}$	[E] Used with log mean mole fraction difference. 23 data points. Average absolute deviation 25%. t_f = formation time.	[141] p. 408
T. Same as 5-21-S, continuous phase coefficient	$\hat{k}_{c,\text{coal}} = 5.959\times 10^{-4}\left(\dfrac{\rho}{M}\right)_{av}$ $\qquad \times \left(\dfrac{D_c}{t_f}\right)^{0.5}\left(\dfrac{\rho_d u_s^3}{g\mu_c}\right)^{0.332}\left(\dfrac{d_p^2 \rho_c \rho_d v_s^3}{\mu_d \sigma g_c}\right)^{0.525}$	[E] Used with log mean mole fraction difference. 20 data points. Average absolute deviation 22%.	[141] p. 409

TABLE 5-21   Mass-Transfer Correlations for Drops, Bubbles, and Bubble Columns (*Continued*)

Conditions	Correlations	Comments E = Empirical, S = Semiempirical, T = Theoretical	References[*]
U. Single liquid drops in gas, gas side coefficient	$$\dfrac{\hat{k}_g M_g d_p P}{D_{gas}\rho_g} = 2 + A N_{Re,g}^{1/2} N_{Sc,g}^{1/3}$$ $A = 0.552$ or 0.60. $$N_{Re,g} = \dfrac{d_p \rho_g v_s}{\mu_g}$$	[E] Used for spray drying (arithmetic partial pressure difference). v_s = slip velocity between drop and gas stream. Sometimes written with $M_g P/\rho_g = RT$.	[90] p. 388 [121]
V. Single water drop in air, liquid side coefficient	$$k_L = 2\left(\dfrac{D_L}{\pi t}\right)^{1/2}, \text{ short contact times}$$ $$k_L = 10\,\dfrac{D_L}{d_p}, \text{ long contact times}$$	[T] Use arithmetic concentration difference. Penetration theory. t = contact time of drop. Gives plot for $k_G a$ also. Air-water system.	[90] p. 389
W. Single bubbles of gas in liquid, continuous phase coefficient, very small bubbles	$$N_{Sh} = \dfrac{k_c' d_b}{D_c} = 1.0(N_{Re}N_{Sc})^{1/3}$$	[T] Solid-sphere Eq. (see Table 5-20-B). $d_b < 0.1$ cm, k_c' is average over entire surface of bubble.	[105] [138] p. 214
X. Same as 5-21-W, medium to large bubbles	$$N_{Sh} = \dfrac{k_c' d_b}{D_c} = 1.13(N_{Re}N_{Sc})^{1/2}$$	[T] Use arithmetic concentration difference. Droplet equation: $d_b > 0.5$ cm.	[138] p. 231
Y. Same as 5-21-X	$$N_{Sh} = \dfrac{k_c' d_b}{D_c} = 1.13(N_{Re}N_{Sc})^{1/2}\left[\dfrac{d_b}{0.45 + 0.2 d_b}\right]$$ $500 \leq N_{Re} \leq 8000$	[S] Use arithmetic concentration difference. Modification of above (X), $d_b > 0.5$ cm. No effect SAA for $d_p > 0.6$ cm.	[83][138] p. 231
Z. Taylor bubbles in single capillaries (square or circular)	$$k_L a = 4.5\left(\dfrac{Du_G}{L_{uc}}\right)^{1/2}\dfrac{1}{d_c}$$ Applicable $\left(\dfrac{u_G + u_L}{L_{slug}}\right)^{0.5} > 3s^{-0.5}$	[E] Air-water L_{uc} = unit cell length, L_{slug} = slug length, d_c = capillary i.d. For most data $k_L a \pm 20\%$.	[153]
AA. Gas-liquid mass transfer in monoliths	$$k_L a \approx 0.1\left(\dfrac{P}{V}\right)^{1/4}$$ P/V = power/volume (kW/m³), range = 100 to 10,000	[E] Each channel in monolith is a capillary. Results are in expected order of magnitude for capillaries based on 5-21-Z. k_L is larger than in stirred tanks.	[93]
AB. Rising small bubbles of gas in liquid, continuous phase. Calderbank and Moo-Young correlation	$$N_{Sh} = \dfrac{k_c' d_b}{D_c} = 2 + 0.31(N_{Gr})^{1/3}N_{Sc}^{1/3},\, d_b < 0.25 \text{ cm}$$ $$N_{Ra} = \dfrac{d_b^3 |\rho_G - \rho_L|g}{\mu_L D_L} = \text{Raleigh number}$$	[E] Use with arithmetic concentration difference. Valid for single bubbles or swarms. Independent of agitation as long as bubble size is constant. Recommended by [136]. Note that $N_{Ra} = N_{Gr}N_{Sc}$.	[47][66] p. 451 [88] p. 119 [152] p. 156 [136]
AC. Same as 5-21-AB, large bubbles	$$N_{Sh} = \dfrac{k_c' d_b}{D_c} = 0.42\,(N_{Gr})^{1/3}N_{Sc}^{1/2},\, d_b > 0.25 \text{ cm}$$ $$\dfrac{\text{Interfacial area}}{\text{volume}} = a = \dfrac{6\,H_g}{d_b}$$	[E] Use with arithmetic concentration difference. For large bubbles, k_c' is independent of bubble size and independent of agitation or liquid velocity. Resistance is entirely in liquid phase for most gas-liquid mass transfer. H_g = fractional gas holdup, volume gas/total volume.	[47][66] p. 452 [88] p. 119 [97] p. 249 [136]
AD. Bubbles in bubble columns. Hughmark correlation	$$N_{Sh} = \dfrac{k_L d}{D} = 2 + b N_{Sc}^{0.546}N_{Re}^{0.779}\left(\dfrac{dg^{1/3}}{D^{2/3}}\right)^{0.116}$$ $b = 0.061$ single gas bubbles; $b = 0.0187$ swarms of bubbles, $$V_s = \dfrac{V_g}{\varphi_G} - \dfrac{V_L}{1-\varphi_G}$$	[E] d = bubble diameter Air-liquid. Recommended by [136, 152]. For swarms, calculate N_{Re} with slip velocity V_s. φ_G = gas holdup V_G = superficial gas velocity Col. diameter = 0.025 to 1.1 m ρ_L' = 776 to 1696 kg/m³ μ_L = 0.0009 to 0.152 Pa·s	[55] [82] [152] p. 144
AE. Bubbles in bubble column	$$k_L a = 0.00315 u_G^{0.59}\mu_{eff}^{-0.84}$$	[E] Recommended by [136].	[57]

TABLE 5-21   Mass-Transfer Correlations for Drops, Bubbles, and Bubble Columns (*Concluded*)

Conditions	Correlations	Comments E = Empirical, S = Semiempirical, T = Theoretical	References[*]
AF. Bubbles in bubble column	$$k_L = \frac{0.15 D}{d_{Vs}} \left(\frac{\nu}{D} \right)^{1/2} N_{Re}^{3/4}$$	[E] d_{Vs} = Sauter mean bubble diameter, $\quad N_{Re} = d_{Vs} u_G \rho_L / \mu_L$. Recommended by [49] based on experiments in $\quad$ industrial system.	[49] [133]
AG. High-pressure bubble column	$k_L a = 1.77 \sigma^{-0.22} \exp(1.65 u_\ell - 65.3 \mu_\ell) \varepsilon_g^{1.2}$ $790 < \rho_L < 1580 \text{ kg/m}^3$ $0.00036 < \mu_\ell < 0.0383 \text{ Pa·s}$ $0.0232 < \sigma_\ell < 0.0726 \text{ N/m}$ $0.028 < u_g < 0.678 \text{ m/s}$ $0 < u_\ell < 0.00089 \text{ m/s}$	[E] Pressure up to 4.24 MPa. T up to 92°C. ε_g = gas holdup. Correlation to estimate ε_g is given. $0.045 < d_{\text{col}} < 0.45 \text{ m}$, $d_{\text{col}}/H_{\text{col}} > 5$ $0.97 < \rho_g < 33.4 \text{ kg/m}^3$	[96]
AH. Three phase (gas-liquid-solid) $\quad$ bubble column to solid spheres	$$N_{sh} = \frac{k_s d_p}{D} = 2.0 + 0.545 N_{Sc}^{1/3} \left(\frac{e d_p^4}{\nu^3} \right)^{0.264}$$ N_{Sc} = 137 to 50,000 (very wide range) d_p = particle diameter (solids)	[E] e = local energy dissipation rate/unit mass, $\quad e = u_g g$ $N_{Sc} = \mu_L / (\rho_L D)$ Recommended by [136].	[129] [136]

See Table 5-22 for agitated systems.
[*]See the beginning of the "Mass Transfer" subsection for references.

TABLE 5-22 Mass-Transfer Correlations for Particles, Drops, and Bubbles in Agitated Systems

Situation	Correlation	Comments E = Empirical, S = Semiempirical, T = Theoretical	References[*]
A. Solid particles suspended in agitated vessel containing vertical baffles, continuous phase coefficient	$\dfrac{k'_{LT}d_p}{D} = 2 + 0.6N_{Re,T}^{1/2}N_{Sc}^{1/3}$ Replace v_{slip} with v_T = terminal velocity. Calculate Stokes' law terminal velocity $v_{Ts} = \dfrac{d_p^2|\rho_p - \rho_c|g}{18\mu_c}$ and correct: $\begin{array}{c\|cccccc} N_{Re,Ts} & 1 & 10 & 100 & 1{,}000 & 10{,}000 & 100{,}000 \\ \hline v_T/v_{Ts} & 0.9 & 0.65 & 0.37 & 0.17 & 0.07 & 0.023 \end{array}$ Approximate: $k'_L = 2k'_{LT}$	[S] Use log mean concentration difference. Modified Frossling equation: $N_{Re,Ts} = \dfrac{v_{Ts}d_p\rho_c}{\mu_c}$ (Reynolds number based on Stokes' law.) $N_{Re,T} = \dfrac{v_T d_p \rho_c}{\mu_c}$ (terminal velocity Reynolds number.) k'_L almost independent of d_p. Harriott suggests different correction procedures. Range k'_L/k'_{LT} is 1.5 to 8.0.	[74][138] p. 220–222 [110] [74]
B. Solid, neutrally buoyant particles, continuous phase coefficient	$N_{Sh} = \dfrac{k'_L d_p}{D} = 2 + 0.47N_{Re,p}^{0.62}N_{Sc}^{0.36}\left(\dfrac{d_{\text{imp}}}{d_{\text{tank}}}\right)^{0.17}$ Graphical comparisons are in Ref. 88, p. 116.	[E] Use log mean concentration difference. Density unimportant if particles are close to neutrally buoyant. Also used for drops. Geometric effect $(d_{\text{imp}}/d_{\text{tank}})$ is usually unimportant. Ref. 102 gives a variety of references on correlations. [E] E = energy dissipation rate per unit mass fluid $= \dfrac{Pg_c}{V_{\text{tank}}\rho_c}$, P = power, $N_{Re,p} = \dfrac{E^{1/3}d_p^{4/3}}{\nu}$	[88] p. 115 [102] p. 132 [152] p. 523
C. Same as 22-B, small particles	$N_{Sh} = 2 + 0.52N_{Re,p}^{0.52}N_{Sc}^{1/3}, N_{Re,p} < 1.0$	[E] Terms same as above.	[88] p. 116
D. Solid particles with significant density difference	$N_{Sh} = \dfrac{k'_L d_p}{D} = 2 + 0.44\left(\dfrac{d_p v_{\text{slip}}}{\nu}\right)^{1/2}N_{Sc}^{0.38}$	[E] Use log mean concentration difference. N_{Sh} standard deviation 11.1%. v_{slip} calculated by methods given in reference.	[102] [110]
E. Small solid particles, gas bubbles or liquid drops, $d_p < 2.5$ mm. Aerated mixing vessels	$N_{Sh} = \dfrac{k'_L d_p}{D} = 2 + 0.31\left[\dfrac{d_p^3|\rho_p - \rho_c|}{\mu_c D}\right]^{1/3}$	[E] Use log mean concentration difference. g = 9.80665 m/s². Second term RHS is free-fall or rise term. For large bubbles, see Table 5-21-AC.	[46][67] p. 487 [97] p. 249
F. Highly agitated systems; solid particles, drops, and bubbles; continuous phase coefficient	$k'_L N_{Sc}^{2/3} = 0.13\left[\dfrac{(P/V_{\text{tank}})\mu_c g_c}{\rho_c^2}\right]^{1/4}$	[E] Use arithmetic concentration difference. Use when gravitational forces overcome by agitation. Up to 60% deviation. Correlation prediction is low (Ref. 102). (P/V_{tank}) = power dissipated by agitator per unit volume liquid.	[47] [66] p. 489 [110]
G. Liquid drops in baffled tank with flat six-blade turbine	$k'_c a = 2.621 \times 10^{-3}\dfrac{(ND)^{1/2}}{d_{\text{imp}}}$ $\times \phi^{0.304}\left(\dfrac{d_{\text{imp}}}{d_{\text{tank}}}\right)^{1.582}N_{Re}^{1.929}N_{Oh}^{1.025}$	[E] Use arithmetic concentration difference. Studied for five systems. $N_{Re} = d_{\text{imp}}^2 N\rho_c/\mu_c$, $N_{Oh} = \mu_c/(\rho_c d_{\text{imp}}\sigma)^{1/2}$ ϕ = volume fraction dispersed phase. N = impeller speed (revolutions/time). For $d_{\text{tank}} = h_{\text{tank}}$, average absolute deviation 23.8%.	[144] p. 437
H. Liquid drops in baffled tank, low volume fraction dispersed phase	$N_{Sh} = \dfrac{k'_c d_p}{D} = 1.237 \times 10^{-5}N_{Sc}^{1/3}N^{2/3}$ $\times N_{Fr}^{5/12}\left(\dfrac{d_{\text{imp}}}{d_p}\right)\left(\dfrac{d_p}{D_{\text{tank}}}\right)^{1/2}\left(\dfrac{\rho_d d_p^2}{\sigma}\right)^{5/4}\phi^{-1/2}$ Stainless steel flat six-blade turbine. Tank had four baffles. Correlation recommended for $\phi \le 0.06$ [Ref. 146] $a = 6\phi/\hat{d}_{32}$, where $\hat{d}_{32}$ is Sauter mean diameter when 33% mass transfer has occurred.	[E] 180 runs, 9 systems, $\phi = 0.01$. k_c is time-averaged. Use arithmetic concentration difference. $N_{Re} = \left(\dfrac{d_{\text{imp}}^2 N_{Sc}}{\mu_c}\right)$, $N_{Fr} = \left(\dfrac{d_{\text{imp}}N^2}{g}\right)$ d_p = particle or drop diameter; σ = interfacial tension, N/m; ϕ = volume fraction dispersed phase; a = interfacial volume, 1/m; and $k_c\alpha D_c^{2/3}$ implies rigid drops. Negligible drop coalescence. Average absolute deviation—19.71%. Graphical comparison given by Ref. 143.	[143] [146] p. 78

TABLE 5-22   Mass-Transfer Correlations for Particles, Drops, and Bubbles in Agitated Systems (*Concluded*)

Situation	Correlation	Comments E = Empirical, S = Semiempirical, T = Theoretical	References[a]
I. Gas bubble swarms in sparged tank reactors	$k'_L a \left(\dfrac{\nu}{g^2}\right)^{1/3} = C \left[\dfrac{P/V_L}{\rho(\nu g^4)^{1/3}}\right]^a \left[\dfrac{q_G}{V_L}\left(\dfrac{\nu}{g^2}\right)^{1/3}\right]^b$ Rushton turbines: $C = 7.94 \times 10^{-4}$, $a = 0.62$, $b = 0.23$. Intermig impellers: $C = 5.89 \times 10^{-4}$, $a = 0.62$, $b = 0.19$.	[E] Use arithmetic concentration difference. Done for biological system, O_2 transfer. $h_{tank}/D_{tank} = 2.1$; P = power, kW. V_L = liquid volume, m^3. q_G = gassing rate, m^3/s. $k'_L a$ = s^{-1}. Since a = m^2/m^3, ν = kinematic viscosity, m^2/s. Low viscosity system. Better fit claimed with q_G/V_L than with u_G (see 5-22-J to N).	[131]
J. Same as 5-22-I	$k'_L a = 2.6 \times 10^{-2}\left(\dfrac{P}{V_L}\right)^{0.4} u_G^{0.5}$	[E] Use arithmetic concentration difference. Ion free water $V_L < 2.6$, u_G = superficial gas velocity in m/s. $500 < P/V_L < 10{,}000$. P/V_L = watts/m^3, V_L = liquid volume, m^3.	[98] [123]
K. Same as 5-22-J	$k'_L a = 2.0 \times 10^{-3}\left(\dfrac{P}{V_L}\right)^{0.7} u_G^{0.2}$	[E] Use arithmetic concentration difference. Water with ions. $0.002 < V_L < 4.4$, $500 < P/V_L < 10{,}000$. Same definitions as 5-22-I.	[98] [101]
L. Same as 5-22-I, baffled tank with standard blade Rushton impeller	$k'_L a = 93.37\left(\dfrac{P}{V_L}\right)^{0.76} u_G^{0.45}$	[E] Air-water. Same definitions as 5-22-I. $0.005 < u_G < 0.025$, $3.83 < N < 8.33$, $400 < P/V_L < 7000$ $h = D_{tank} = 0.305$ or 0.610 m. V_G = gas volume, m^3, N = stirrer speed, rpm. Method assumes perfect liquid mixing.	[67] [98]
M. Same as 5-22-L	$k'_L a \dfrac{d_{imp}^2}{D} = 7.57\left[\dfrac{\mu_{eff}}{\rho D}\right]^{0.5}\left[\dfrac{\mu_G}{\mu_{eff}}\right]^{0.694}$ $\times \left[\dfrac{d_{imp}^2 N \rho_L}{\mu_{eff}}\right]^{1.11}\left(\dfrac{u_G d}{\sigma}\right)^{0.447}$ d_{imp} = impeller diameter, m; D = diffusivity, m^2/s	[E] Use arithmetic concentration difference. CO_2 into aqueous carboxyl polymethylene. Same definitions as 5-22-L. μ_{eff} = effective viscosity from power law model, Pa·s. σ = surface tension liquid, N/m.	[98] [115]
N. Same as 5-22-L, bubbles	$\dfrac{k'_L a d_{imp}^2}{D} = 0.060\left(\dfrac{d_{imp}^2 N \rho}{\mu_{eff}}\right)\left(\dfrac{d_{imp}^2 N^2}{g}\right)^{0.19}\left(\dfrac{\mu_{eff} u_G}{\sigma}\right)^{0.6}$	[E] Use arithmetic concentration difference. O_2 into aqueous glycerol solutions. O_2 into aqueous millet jelly solutions. Same definitions as 5-22-L.	[98] [160]
O. Gas bubble swarm in sparged stirred tank reactor with solids present	$\dfrac{k'_L a}{(k'_L a)_o} = 1 - 3.54(\varepsilon_s - 0.03)$ $300 \le P/V_{rx} < 10{,}000$ W/m^3, $0.03 \le \varepsilon_s \le 0.12$ $0.34 \le u_G \le 4.2$ cm/s, $5 < \mu_L < 75$ Pa·s	[E] Use arithmetic concentration difference. Solids are glass beads, d_p = 320 μm. ε_s = solids holdup m^3/m^3 liquid, $(k'_L a)_o$ = mass transfer in absence of solids. Ionic salt solution—noncoalescing.	[38] [132]
P. Surface aerators for air-water contact	$\dfrac{k_L a}{N} = b N_p^{0.71} N_{Fr}^{0.48} N_{Re}^{0.82}\left(\dfrac{H}{d}\right)^{-0.54}\left(\dfrac{V}{d^3}\right)^{-1.08}$ $b = 7 \times 10^{-6}$, $N_p = P/(\rho N^3 d^5)$ $N_{Re} = N d^2 \rho_{liq}/\mu_{liq}$ $N_{Fr} = N^2 d/g$, P/V = 90 to 400 W/m^3	[E] Three impellers: Pitched blade downflow turbine, pitched blade upflow turbine, standard disk turbine. Baffled cylindrical tanks 1.0- and 1.5-m ID and 8.2×8.2-m square tank. Submergence optimized all cases. Good agreement with data. N = impeller speed, s^{-1}; d = impeller diameter, m; H = liquid height, m; V = liquid volume, m^3; $k_L a$ = s^{-1}, g = acceleration gravity = 9.81 m/s^2	[113]
Q. Gas-inducing impeller for air-water contact	$k_L a V(\nu/g^2)^{1/3} d^3 = A N_{Fr}^B \left(\dfrac{V_A}{V}\right)^C$ Single impeller: $A = 0.00497$, $B = 0.56$, $C = 0.32$ Multiple impeller: $A = 0.00746$, $B = 0.54$, $C = 0.38$	[E] Same tanks and same definitions as in 5-22-P. V_A = active volume = $p/(\pi \rho g N d)$.	[113]
R. Gas-inducing impeller with dense solids	$Sh_{GL} = \dfrac{k_L a d_{st}^2}{D} = (1.26 \times 10^{-5}) N_{Re}^{1.8} N_{Sc}^{0.9} N_{We}^{-0.1}$ $N_{Re} = \rho N d_{St}^2/\mu$, $N_{Sc} = \mu/(\rho D)$, $N_{We} = \rho N^2 d_{St}^3/\sigma$	[E] Hydrogenation with Raney-type nickel catalyst in stirred autoclave. Used varying T, p, solvents. d_{st} = stirrer diameter.	[78]

See also Table 5-21.

[a]See the beginning of the "Mass Transfer" subsection for references.

TABLE 5-23   Mass-Transfer Correlations for Fixed and Fluidized Beds

Transfer is to or from particles

Situation	Correlation	Comments E = Empirical, S = Semiempirical, T = Theoretical	References[a]
A. For gases, fixed and fluidized beds, Gupta and Thodos correlation	$j_H = j_D = \dfrac{2.06}{\varepsilon N_{Re}^{0.575}}$, $90 \le N_{Re} \le A$ Equivalent: $N_{Sh} = \dfrac{2.06}{\varepsilon} N_{Re}^{0.425} N_{Sc}^{1/3}$ For other shapes: $\dfrac{\varepsilon j_D}{(\varepsilon j_D)_{sphere}} = 0.79$ (cylinder) or 0.71 (cube)	[E] For spheres. $N_{Re} = \dfrac{v_{super} d_p \rho}{\mu}$ $A = 2453$ [Ref. 141], $A = 4000$ [Ref. 77]. For $N_{Re} > 1900$, $j_H = 1.05 j_D$. Heat transfer result is in absence of radiation. $N_{Sh} = \dfrac{k' d_s}{D}$ Graphical results are available for N_{Re} from 1900 to 10,300. $a = \dfrac{\text{surface area}}{\text{volume}} = 6(1 - \varepsilon)/d_p$ For spheres, d_p = diameter. For nonspherical: $d_p = 0.567 \sqrt{\text{Part. Surf. Area}}$	[72] [73] [77] p. 195 [141]
B. For gases, for fixed beds, Petrovic and Thodos correlation	$N_{Sh} = \dfrac{0.357}{\varepsilon} N_{Re}^{0.641} N_{Sc}^{1/3}$ $3 < N_{Re} < 900$ can be extrapolated to $N_{Re} < 2000$.	[E] Packed spheres, deep beds. Corrected for axial dispersion with axial Peclet number = 2.0. Prediction is low at low N_{Re}. N_{Re} defined as in 5-23-A.	[116][128] p. 214 [155]
C. For gases and liquids, fixed and fluidized beds	$j_D = \dfrac{0.4548}{\varepsilon N_{Re}^{0.4069}}$, $10 \le N_{Re} \le 2000$ $j_D = \dfrac{N_{Sh}}{N_{Re} N_{Sc}^{1/3}}$, $N_{Sh} = \dfrac{k' d_s}{D}$	[E] Packed spheres, deep bed. Average deviation $\pm 20\%$, $N_{Re} = d_p v_{super} \rho/\mu$. Can use for fluidized beds. $10 \le N_{Re} \le 4000$.	[60][66] p. 484
D. For gases, fixed beds	$j_D = \dfrac{0.499}{\varepsilon N_{Re}^{0.382}}$	[E] Data on sublimation of naphthalene spheres dispersed in inert beds. $0.1 < N_{Re} < 100$, $N_{Sc} = 2.57$. Correlation coefficient = 0.978.	[80]
E. For liquids, fixed bed, Wilson and Geankoplis correlation	$j_D = \dfrac{1.09}{\varepsilon N_{Re}^{2/3}}$, $0.0016 < N_{Re} < 55$ $165 \le N_{Sc} \le 70{,}600$, $0.35 < \varepsilon < 0.75$ Equivalent: $N_{Sh} = \dfrac{1.09}{\varepsilon} N_{Re}^{1/3} N_{Sc}^{1/3}$ $j_D = \dfrac{0.25}{\varepsilon N_{Re}^{0.31}}$, $55 < N_{Re} < 1500$, $165 \le N_{Sc} \le 10{,}690$ Equivalent: $N_{Sh} = \dfrac{0.25}{\varepsilon} N_{Re}^{0.69} N_{Sc}^{1/3}$	[E] Beds of spheres, $N_{Re} = \dfrac{d_p V_{super} \rho}{\mu}$ Deep beds. $N_{Sh} = \dfrac{k' d_s}{D}$	[66] p. 484 [77] p. 195 [141] p. 287 [158]
F. For liquids, fixed beds, Ohashi et al. correlation	$N_{Sh} = \dfrac{k' d_s}{D} = 2 + 0.51 \left(\dfrac{E^{1/3} d_p^{4/3} \rho}{\mu} \right)^{0.60} N_{Sc}^{1/3}$ E = Energy dissipation rate per unit mass of fluid $= 50(1 - \varepsilon)\varepsilon^2 C_{Do} \left(\dfrac{v_r^3}{d_p} \right)$, m²/s³ $= \left[\dfrac{50(1 - \varepsilon)C_D}{\varepsilon} \right] \left(\dfrac{v_{super}^3}{d_p} \right)$ General form: $N_{Sh} = 2 + K \left(\dfrac{E^{1/3} D_p^{4/3} \rho}{\mu} \right)^{\alpha} N_{Sc}^{\beta}$ applies to single particles, packed beds, two-phase tube flow, suspended bubble columns, and stirred tanks with different definitions of E.	[S] Correlates large amount of published data. Compares number of correlations, v_r = relative velocity, m/s. In packed bed, $v_r = v_{super}/\varepsilon$. C_{Do} = single particle drag coefficient at v_{super} calculated from $C_{Do} = A N_{Re_t}^{-m}$. <table><tr><td>N_{Re}</td><td>A</td><td>m</td></tr><tr><td>0 to 5.8</td><td>24</td><td>1.0</td></tr><tr><td>5.8 to 500</td><td>10</td><td>0.5</td></tr><tr><td>>500</td><td>0.44</td><td>0</td></tr></table> Ranges for packed bed: $0.001 < N_{Re} < 1000$, $505 < N_{Sc} < 70{,}600$, $0.2 < \dfrac{E^{1/3} d_p^{4/3} \rho}{\mu} < 4600$ Compares different situations versus general correlation. See also 5-20-F.	[108]

TABLE 5-23   Mass Transfer Correlations for Fixed and Fluidized Beds (*Continued*)

Situation	Correlation	Comments E = Empirical, S = Semiempirical, T = Theoretical	References*
G. Electrolytic system. Pall rings. Transfer from fluid to rings.	Full liquid upflow: $N_{Sh} = k_L d_e/D = 4.1 N_{Re}^{0.39} N_{Sc}^{1/3}$ $N_{Re} d_e u/\nu = 80$ to 550 Irrigated liquid downflow (no gas flow): $N_{Sh} = 5.1 N_{Re}^{0.44} N_{Sc}^{1/3}$	[E] d_e = diameter of sphere with same surface area as Pall ring. Full liquid upflow agreed with literature values. Schmidt number dependence was assumed from literature values. In downflow, N_{Re} used superficial fluid velocity.	[69]
H. For liquids, fixed and fluidized beds	$\varepsilon j_D = \dfrac{1.1068}{N_{Re}^{0.72}}$, $1.0 < N_{Re} \leq 10$ $\varepsilon j_D = \dfrac{N_{Sh}}{N_{Re} N_{Sc}^{1/3}}$, $N_{Sh} = \dfrac{k' d_s}{D}$	[E] Spheres: $N_{Re} = \dfrac{d_p v_{super} \rho}{\mu}$	[59][66] p. 484
I. For gases and liquids, fixed and fluidized beds, Dwivedi and Upadhyay correlation	$\varepsilon j_D = \dfrac{0.765}{N_{Re}^{0.82}} + \dfrac{0.365}{N_{Re}^{0.386}}$ Gases: $10 \leq N_{Re} \leq 15{,}000$. Liquids: $0.01 \leq N_{Re} \leq 15{,}000$. $N_{Re} = \dfrac{d_p v_{super} \rho}{\mu}$, $N_{Sh} = \dfrac{k' d_s}{D}$	[E] Deep beds of spheres, $j_D = \dfrac{N_{Sh}}{N_{Re} N_{Sc}^{1/3}}$ Best fit correlation at low conc. [52] Based on 20 gas studies and 17 liquid studies. Recommended instead of 5-23-C or E.	[59] [77] p. 196 [52]
J. For gases and liquids, fixed bed	$j_D = 1.17 N_{Re}^{-0.415}$, $10 \leq N_{Re} \leq 2500$ $j_D = \dfrac{k'}{v_{av}} \dfrac{p_{BM}}{P} N_{Sc}^{2/3}$	[E] Spheres: Variation in packing that changes ε not allowed for. Extensive data referenced. $0.5 < N_{Sc} < 15{,}000$. Comparison with other results are shown. $N_{Re} = \dfrac{d_p v_{super} \rho}{\mu}$	[138] p. 241
K. For liquids, fixed and fluidized beds, Rahman and Streat correlation	$N_{Sh} = \dfrac{0.86}{\varepsilon} N_{Re} N_{Sc}^{1/3}$, $2 \leq N_{Re} \leq 25$	[E] Can be extrapolated to $N_{Re} = 2000$. $N_{Re} = d_p v_{super} \rho/\mu$. Done for neutralization of ion exchange resin.	[119]
L. Size exclusion chromatography of proteins	$N_{Sh} = \dfrac{k_L d}{D} = \dfrac{1.903}{\varepsilon} N_{Re}^{1/3} N_{Sc}^{1/3}$	[E] Slow mass transfer with large molecules. Aqueous solutions. Modest increase in N_{Sh} with increasing velocity.	[79]
M. Liquid-free convection with fixed bed Raschig rings. Electrochemical.	$N_{Sh} = kd/D = 0.15 (N_{Sc} N_{Gr})^{0.32}$ $N_{Gr} = $ Grashof no. $= gd^3 \Delta\rho/(\nu^2 \rho)$ If forced convection superimposed, $N_{Sh, overall} = (N_{Sh,forced}^3 + N_{Sh,free}^3)^{1/3}$	[E] d = Raschig ring diameter, h = bed height $1810 < N_{Sc} < 2532$, $0.17 < d/h < 1.0$ $10.6 \times 10^6 < N_{Sc} N_{Gr} < 21 \times 10^7$	[135]
N. Oscillating bed packed with Raschig rings. Dissolution of copper rings.	Batch (no net solution flow): $N_{Sh} = 0.76 N_{Sc}^{0.33} N_{Re,v}^{0.7} (d_c/h)^{0.35}$ $503 < N_{Re,v} < 2892$ $960 < N_{Sc} < 1364$, $2.3 < d_c/h < 7.6$	[E] $N_{Sh} = kd_c/D$, $N_{Re,v}$ = vibrational Re $= \rho v_v d_c/\mu$ v_v = vibrational velocity (intensity) d_c = col. diameter, h = column height Average deviation is ± 12%.	[61]
O. For liquids and gases, Ranz and Marshall correlation	$N_{Sh} = \dfrac{k'd}{D} = 2.0 + 0.6 N_{Sc}^{1/3} N_{Re}^{1/2}$ $N_{Re} = \dfrac{d_p v_{super} \rho}{\mu}$	[E] Based on freely falling, evaporating spheres (see 5-20-C). Has been applied to packed beds, prediction is low compared to experimental data. Limit of 2.0 at low N_{Re} is too high. Not corrected for axial dispersion.	[121][128] p. 214 [155] [110]
P. For liquids and gases, Wakao and Funazkri correlation	$N_{Sh} = 2.0 + 1.1 N_{Sc}^{1/3} N_{Re}^{0.6}$, $3 < N_{Re} < 10{,}000$ $N_{Sh} = \dfrac{k'_{film} d_p}{D}$, $N_{Re} = \dfrac{\rho_f v_{super} \rho}{\mu}$ $\dfrac{\varepsilon D_{axial}}{D} = 10 + 0.5 N_{Sc} N_{Re}$	[E] Correlate 20 gas studies and 16 liquid studies. Corrected for axial dispersion with: Graphical comparison with data shown [128], p. 215, and [155]. D_{axial} is axial dispersion coefficient.	[128] p. 214 [155]
Q. Acid dissolution of limestone in fixed bed	$N_{Sh} = 1.77 N_{Re}^{0.56} N_{Sc}^{1/3} (1 - \varepsilon)^{0.44}$ $20 < N_{Re} < 6000$	[E] Best fit was to correlation of Chu et al., *Chem. Eng. Prog.*, **49**(3), 141(1953), even though no reaction in original.	[94]
R. Semifluidized or expanded bed. Liquid-solid transfer.	$N_{Sh} = \dfrac{k_{film} d_p}{D} = 2 + 1.5 (1 - \varepsilon_L) N_{Re}^{1/3} N_{Sc}^{1/3}$ $N_{Re} = \rho_p d_p u/\mu \varepsilon_L$; $N_{Sc} = \mu/\rho D$	[E] ε_L = liquid-phase void fraction, ρ_p = particle density, ρ = fluid density, d_p = particle diameter. Fits expanded bed chromatography in viscous liquids.	[64] [159]

TABLE 5-23   Mass Transfer Correlations for Fixed and Fluidized Beds (*Concluded*)

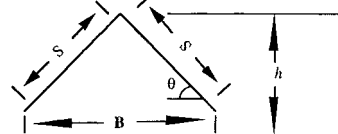| Situation	Correlation	Comments E = Empirical, S = Semiempirical, T = Theoretical	References*
S. Mass-transfer structured packing and static mixers. Liquid with or without fluidized particles. Electrochemical	Fixed bed: $j' = 0.927N_{Re'}^{0.572}$, $N'_{Re} < 219$ $j' = 0.443N_{Re'}^{-0.435}$, $219 < N'_{Re} < 1360$ Fluidized bed with particles: $j = 6.02N_{Re}^{-0.885}$, or $j' = 16.40N_{Re'}^{-0.950}$ Natural convection: $N_{Sh} = 0.252(N_{Sc}N_{Gr})^{0.299}$ Bubble columns: Structured packing: $N_{St} = 0.105(N_{Re}N_{Fr}N_{Sc}^2)^{-0.268}$ Static mixer: $N_{St} = 0.157(N_{Re}N_{Fr}N_{Sc}^2)^{-0.298}$	[E] Sulzer packings, $j' = \dfrac{k\cos\beta}{v}N_{Sc}^{2/3}$, β = corrugation incline angle. $N_{Re'} = v'\,d'_h\rho/\mu$, $v' = v_{super}/(\varepsilon\cos\beta)$, d'_h = channel side width. Particles enhance mass transfer in laminar flow for natural convection. Good fit with correlation of Ray et al., *Intl. J. Heat Mass Transfer*, **41**, 1693 (1998). $N_{Gr} = g\,\Delta\,\rho Z^3\rho/\mu^2$, Z = corrugated plate length. Bubble column results fit correlation of Neme et al., *Chem. Eng. Technol.*, **20**, 297 (1997) for structured packing. N_{St} = Stanton number = kZ/D N_{Fr} = Froude number = v_{super}^2/gz	[48]
T. Liquid fluidized beds	$N_{Sh} = \dfrac{2\xi/\varepsilon^m + \left[\dfrac{(2\xi/\varepsilon^m)(1-\varepsilon)^{1/2}}{[1-(1-\varepsilon)^{1/3}]^2} - 2\right]\tan h\,(\xi/\varepsilon^m)}{\dfrac{\xi/\varepsilon^m}{1-(1-\varepsilon^{1/2})} - \tan h\,(\xi/\varepsilon^m)}$ where $\xi = \left[\dfrac{1}{(1-\varepsilon)^{1/3}} - 1\right]\dfrac{\alpha}{2}N_{Sc}^{1/3}N_{Re}^{1/2}$ This simplifies to: $N_{Sh} = \dfrac{\varepsilon^{1-2m}}{(1-\varepsilon)^{1/3}}\left[\dfrac{1}{(1-\varepsilon)^{1/3}} - 1\right]\dfrac{\alpha^2}{2}N_{Re}N_{Sc}^{2/3}$ $(N_{Re} < 0.1)$	[S] Modification of theory to fit experimental data. For spheres, $m = 1$, $N_{Re} > 2$. $N_{Sh} = \dfrac{k'_L d_p}{D}$, $N_{Re} = \dfrac{V_{super}d_p\xi}{\mu}$ $m = 1$ for $N_{Re} > 2$; $m = 0.5$ for $N_{Re} < 1.0$; ε = voidage; α = const. Best fit data is $\alpha = 0.7$. Comparison of theory and experimental ion exchange results in Ref. 92.	[92] [106] [125]
U. Liquid fluidized beds	$N_{Sh} = 0.250N_{Re}^{0.023}N_{Ga}^{0.306}\left(\dfrac{\rho_s-\rho}{\rho}\right)^{0.282}N_{Sc}^{0.410}$ $(\varepsilon < 0.85)$ $N_{Sh} = 0.304N_{Re}^{-0.057}N_{Ga}^{0.332}\left(\dfrac{\rho_s-\rho}{\rho}\right)^{0.297}N_{Sc}^{0.404}$ $(\varepsilon > 0.85)$ This can be simplified (with slight loss in accuracy at high ε) to $N_{Sh} = 0.245N_{Ga}^{0.323}\left(\dfrac{\rho_s-\rho}{\rho}\right)^{0.300}N_{Sc}^{0.400}$	[E] Correlate amount of data from literature. Predicts very little dependence of N_{Sh} on velocity. Compare large number of published correlations. $N_{Sh} = \dfrac{k'_L d_p}{D}$, $N_{Re} = \dfrac{d_p\rho v_{super}}{\mu}$, $N_{Ga} = \dfrac{d_p^3\rho^2 g}{\mu^2}$, $N_{Sc} = \dfrac{\mu}{\rho D}$ $1.6 < N_{Re} < 1320$, $2470 < N_{Ga} < 4.42\times10^6$ $0.27 < \dfrac{\rho_s-\rho}{\rho} < 1.114$, $305 < N_{Sc} < 1595$	[151]
V. Liquid film flowing over solid particles with air present, trickle bed reactors, fixed bed	$N_{Sh} = \dfrac{k_L}{aD} = 1.8N_{Re}^{1/2}N_{Sc}^{1/3}$, $0.013 < N_{Re} < 12.6$ two-phases, liquid trickle, no forced flow of gas. $N_{Sh} = 0.8N_{Re}^{1/2}N_{Sc}^{1/3}$, one-phase, liquid only.	[E] $N_{Re} = \dfrac{L}{a\mu}$, irregular granules of benzoic acid, $0.29 \leq d_p \leq 1.45$ cm. L = superficial liquid flow rate, kg/m²s. a = surface area/col. volume, m²/m³.	[130]
W. Supercritical fluids in packed bed	$\dfrac{N_{Sh}}{(N_{Sc}N_{Gr})^{1/4}} = 0.5265\left(\dfrac{(N_{Re}^{1/2}N_{Sc}^{1/3})}{(N_{Sc}N_{Gr})^{1/4}}\right)^{1.6808}$ $+ 2.48\left\|\left(\dfrac{N_{Re}^2 N_{Sc}^{1/3}}{N_{Gr}}\right)^{0.6439} - 0.8768\right\|^{1.553}$	[E] Natural and forced convection. $0.3 < N_{Re} < 135$.	[99]
X. Cocurrent gas-liquid flow in fixed beds	Downflow in trickle bed and upflow in bubble columns.	Literature review and meta-analysis. Analyzed both downflow and upflow. Recommendations for best mass- and heat-transfer correlations (see reference).	[95]
Y. Liquid-solid transfer. Electrochemical reaction. Lessing rings. Transfer from liquid to solid	Liquid only: $N_{Sh} = kd/D = 1.57N_{Sc}^{1/3}N_{Re}^{0.46}$ $1390 < N_{Sc} < 4760$, $166 < N_{Re} < 722$ Cocurrent two-phase (liquid and gas) in packed bubble column: $N_{Sh} = 1.93N_{Sc}^{1/3}\,N_{Re}^{0.34}\,N_{Re,gas}^{0.11}$ $60 < N_{Re,gas} < 818$, $144 < N_{Re} < 748$	[E] Electrochemical reactors only. d = Lessing ring diameter, $1 < d < 1.4$ cm, $N_{Re} = \rho v_{super}d/\mu$, Deviation ±7% for both cases. $N_{Re,gas} = \rho_{gas}V_{super,gas}d/\mu_{gas}$ Presence of gas enhances mass transfer.	[75]

NOTE: For $N_{Re} < 3$ convective contributions which are not included may become important. Use with logarithmic concentration difference (integrated form) or with arithmetic concentration difference (differential form).

*See the beginning of the "Mass Transfer" subsection for references.

TABLE 5-24   Mass-Transfer Correlations for Packed Two-Phase Contactors—Absorption, Distillation, Cooling Towers, and Extractors (Packing Is Inert)

Situation	Correlations	Comments E = Empirical, S = Semiempirical, T = Theoretical	References[*]
A. Absorption, counter-current, liquid-phase coefficient H_L, Sherwood and Holloway correlation for random packings	$H_L = a_L \left(\dfrac{L}{\mu_L}\right)^n N_{Sc,L}^{0.5}$, $L = $ lb/hr ft^2 Ranges for 5-24-B (G and L) Packing a_G b c G L a_L n Raschig rings 3/8 inch 2.32 0.45 0.47 200–500 500–1500 0.00182 0.46 1 7.00 0.39 0.58 200–800 400–500 0.010 0.22 1 6.41 0.32 0.51 200–600 500–4500 0.22 2 3.82 0.41 0.45 200–800 500–4500 0.0125 0.22 Berl saddles 1/2 inch 32.4 0.30 0.74 200–700 500–1500 0.0067 0.28 1/2 0.811 0.30 0.24 200–800 400–4500 1 1.97 0.36 0.40 200–800 400–4500 0.0059 0.28 1.5 5.05 0.32 0.45 200–1000 400–4500 0.0062 0.28	[E] From experiments on desorption of sparingly soluble gases from water. Graphs [Ref. 138], p. 606. Equation is dimensional. A typical value of n is 0.3 [Ref. 66] has constants in kg, m, and s units for use in 5-24-A and B with $\hat{k}_G$ in kgmole/s m^2 and $\hat{k}_L$ in kgmole/s m^2 (kgmol/m^3). Constants for other packings are given by Refs. 104, p. 187 and 152, p. 239. $H_L = \dfrac{L_M}{\hat{k}_L a}$ $L_M = $ lbmol/hr ft^2, $\hat{k}_L = $ lbmol/hr ft^2, $a = $ ft^2/ft^3, μ_L in lb/(hr ft). Range for 5-24-A is $400 < L < 15{,}000$ lb/hr ft^2	[104] p. 187 [105] [138] p. 606 [157] [156]
B. Absorption counter-current, gas-phase coefficient H_G, for random packing	$H_G = \dfrac{G_M}{\hat{k}_G a} = \dfrac{a_G (G)^b N_{Sc,v}^{0.5}}{(L)^c}$	[E] Based on ammonia-water-air data in Fellinger's 1941 MIT thesis. Curves: Refs. 104, p. 186 and 138, p. 607. Constants given in 5-24-A. The equation is dimensional. $G = $ lb/hr ft^2, $G_M = $ lbmol/hr ft^2, $\hat{k}_G = $ lbmol/hr ft^2.	[104] p. 189 [138] p. 607 [157]
C. Absorption and and distillation, counter-current, gas and liquid individual coefficients and wetted surface area, Onda et al. correlation for random packings	$\dfrac{k_G' RT}{a_p D_G} = A\left(\dfrac{G}{a_p \mu_G}\right)^{0.7} N_{Sc,G}^{1/3} (a_p d_p')^{-2.0}$ $k_L'\left(\dfrac{\rho_L}{\mu_L g}\right)^{1/3} = 0.0051 \left(\dfrac{L}{a_w \mu_L}\right)^{2/3} N_{Sc,L}^{-1/2} (a_p d_p')^{0.4}$ $k_L' = $ lbmol/hr ft^2 (lbmol/ft^3) [kgmol/s m^2 (kgmol/m^3)] $\dfrac{a_w}{a_p} = 1 - \exp\left\{ \begin{array}{l} -1.45\left(\dfrac{\sigma_c}{\sigma}\right)^{0.75}\left(\dfrac{L}{a_p \mu_L}\right)^{0.1} \\ \times \left(\dfrac{L^2 a_p}{\rho_L^2 g}\right)^{-0.05}\left(\dfrac{L}{\rho_L \sigma a_p}\right)^{0.2} \end{array}\right\}$	[E] Gas absorption and desorption from water and organics plus vaporization of pure liquids for Raschig rings, saddles, spheres, and rods. $d_p' = $ nominal packing size, $a_p = $ dry packing surface area/volume, $a_w = $ wetted packing surface area/volume. Equations are dimensionally consistent, so any set of consistent units can be used. $\sigma = $ surface tension, dynes/cm. $A = 5.23$ for packing $\geq$ 1/2 inch (0.012 m) $A = 2.0$ for packing < 1/2 inch (0.012 m) $k_G' = $ lbmol/hr ft^2 atm [kg mol/s m^2 (N/m^2)] Critical surface tensions, $\sigma_C = 61$ (ceramic), 75 (steel), 33 (polyethylene), 40 (PVC), 56 (carbon) dynes/cm. $4 < \dfrac{L}{a_w \mu_L} < 400$ $5 < \dfrac{G}{a_p \mu_G} < 1000$ Most data $\pm$ 20% of correlation, some $\pm$ 50%. Graphical comparison with data in Ref. 109.	[44] [90] p. 380 [109][149] p. 355 [156]
D. Distillation and absorption, counter-current, random packings, modification of Onda correlation, Bravo and Fair correlation to determine interfacial area	Use Onda's correlations (5-24-C) for k_G' and k_L'. Calculate: $H_G = \dfrac{G}{k_G' a_e P M_G}$, $H_L = \dfrac{L}{k_L' a_e \rho_L}$, $H_{OG} = H_G + \lambda H_L$ $\lambda = \dfrac{m}{L_M/G_M}$ $a_e = 0.498 a_p \left(\dfrac{\sigma^{0.5}}{Z^{0.4}}\right)(N_{Ca,L} N_{Re,G})^{0.392}$ $N_{Re,G} = \dfrac{6G}{a_p \mu_G}$, $N_{Ca,L} = \dfrac{L\mu_L}{\rho_L \sigma g_c}$ (dimensionless)	[E] Use Bolles & Fair (Ref. 43) database to determine new effective area a_e to use with Onda et al. (Ref. 109) correlation. Same definitions as 5-24-C. $P = $ total pressure, atm; $M_G = $ gas, molecular weight; $m = $ local slope of equilibrium curve; $L_M/G_M = $ slope operating line; $Z = $ height of packing in feet. Equation for a_e is dimensional. Fit to data for effective area quite good for distillation. Good for absorption at low values of $(N_{Ca,L} \times N_{Re,G})$, but correlation is too high at higher values of $(N_{Ca,L} \times N_{Re,G})$.	[44]

TABLE 5-24   Mass-Transfer Correlations for Packed Two-Phase Contactors—Absorption, Distillation, Cooling Towers, and Extractors (Packing Is Inert) (*Continued*)

Situation	Correlations	Comments E = Empirical, S = Semiempirical, T = Theoretical	References[a]
E. Absorption and distillation, countercurrent gas-liquid flow, random and structured packing. Determine H_L and H_G	$H_G = \left(\dfrac{0.226}{f_p}\right)\left(\dfrac{N_{Sc}}{0.660}\right)^b \left(\dfrac{G_x}{6.782}\right)^{-0.5}\left(\dfrac{G_y}{0.678}\right)^{0.35}$ $H_L = \left(\dfrac{0.357}{f_P}\right)\left(\dfrac{N_{Sc}}{372}\right)^{0.5}\left(\dfrac{G_x/\mu}{6.782/0.0008937}\right)^{0.3}$ Relative transfer coefficients [91], f_p values are in table: 	Size, in.	Ceramic Raschig rings	Ceramic Berl saddles	Metal Pall rings	Metal Intalox	Metal Hypac
---	---	---	---	---	---
0.5	1.52	1.58	—	—	—
1.0	1.20	1.36	1.61	1.78	1.51
1.5	1.00	—	1.34	—	—
2.0	0.85	—	1.14	1.27	1.07 | Norton Intalox structured: 2T, f_p = 1.98; 3T, f_p = 1.94.	[S] H_G based on NH_3 absorption data (5–28B) for which $H_{G,\text{base}}$ = 0.226 m with $N_{Sc,\text{base}}$ = 0.660 at $G_{x,\text{base}}$ = 6.782 kg/(s m^2) and $G_{y,\text{base}}$ = 0.678 kg/(s m^2) with 1½ in. ceramic Raschig rings. The exponent b on N_{Sc} is reported as either 0.5 or as ⅔. $f_p = \dfrac{H_G \text{ for } NH_3 \text{ with } 1½ \text{ Raschig rings}}{H_G \text{ for } NH_3 \text{ with desired packing}}$ H_L based on O_2 desorption data (5-24-A). Base viscosity, μ_{base} = 0.0008937 kg/(m s). H_L in m. G_y < 0.949 kg/(s m^2), 0.678 < G_x < 6.782 kg/(s m^2). Best use is for absorption and stripping. Limited use for organic distillation [156].	[66] p. 686, 659 [138] [156]
F. Absorption, cocurrent downward flow, random packings, Reiss correlation	Air-oxygen-water results correlated by $k'_L a$ = $0.12 E_L^{0.5}$. Extended to other systems. $k'_L a = 0.12 E_L^{0.5}\left(\dfrac{D_L}{2.4 \times 10^5}\right)^{0.5}$ $E_L = \left(\dfrac{\Delta p}{\Delta L}\right)_{\text{2-phase}} v_L$ $\dfrac{\Delta p}{\Delta L}$ = pressure loss in two-phase flow = lbf/ft^2 ft $k'_G a = 2.0 + 0.91 E_G^{2/3}$ for NH_3 $E_g = \left(\dfrac{\Delta p}{\Delta L}\right)_{\text{2-phase}} v_g$ v_g = superficial gas velocity, ft/s	[E] Based on oxygen transfer from water to air at 77°F. Liquid film resistance controls. (D_{water} @ 77°F = 2.4×10^{-5}). Equation is dimensional. Data was for thin-walled polyethylene Raschig rings. Correlation also fit data for spheres. Fit ±25%. See [122] for graph. $k'_L a$ = s^{-1} D_L = cm/s E_L = ft, lbf/s ft^3 v_L = superficial liquid velocity, ft/s [E] Ammonia absorption into water from air at 70°F. Gas-film resistance controls. Thin-walled polyethylene Raschig rings and 1-inch Intalox saddles. Fit ±25%. See [122] for fit. Terms defined as above.	[122] [130] p. 217 [122]
G. Absorption, stripping, distillation, counter-current, H_L, and H_G, random packings, Bolles and Fair correlation	For Raschig rings, Berl saddles, and spiral tile: $H_L = \dfrac{\phi C_{\text{flood}}}{3.28} N_{Sc,L}^{0.5}\left(\dfrac{Z}{3.05}\right)^{0.15}$ C_{flood} = 1.0 if below 40% flood—otherwise, use figure in [54] and [157]. $H_G = \dfrac{A\psi(d'_{\text{col}})^m Z^{0.33} N_{Sc,G}^{0.5}}{\left[L\left(\dfrac{\mu_L}{\mu_{\text{water}}}\right)^{0.16}\left(\dfrac{\rho_{\text{water}}}{\rho_L}\right)^{1.25}\left(\dfrac{\sigma_{\text{water}}}{\sigma_L}\right)^{0.8}\right]^n}$ Figures for ϕ and ψ in [42 and 43] Ranges: 0.02 < ϕ > 0.300; 25 < ψ < 190 m.	[E] Z = packed height, m of each section with its own liquid distribution. The original work is reported in English units. Cornell et al. (Ref. 54) review early literature. Improved fit of Cornell's ϕ values given by Bolles and Fair (Refs. [42], [43]) and [157]. A = 0.017 (rings) or 0.029 (saddles) d'_{col} = column diameter in m (if diameter > 0.6 m, use d'_{col} = 0.6) m = 1.24 (rings) or 1.11 (saddles) n = 0.6 (rings) or 0.5 (saddles) L = liquid rate, kg/(s m^2), μ_{water} = 1.0 Pa·s, ρ_{water} = 1000 kg/m^3, σ_{water} = 72.8 mN/m (72.8 dyn/cm). H_G and H_L will vary from location to location. Design each section of packing separately.	[42, 43, 54] [77] p. 428 [90] p. 381 [141] p. 353 [157] [156]
H. Distillation and absorption. Counter-current flow. Structured packings. Gauze-type with triangular flow channels, Bravo, Rocha, and Fair correlation	Equivalent channel: $d_{eq} = Bh\left[\dfrac{1}{B + 2S} + \dfrac{1}{2S}\right]$ Use modified correlation for wetted wall column (See 5-18-F) $N_{Sh,v} = \dfrac{k'_v d_{eq}}{D_v} = 0.0338 N_{Re,v}^{0.8} N_{Sc,v}^{0.333}$ $N_{Re,v} = \dfrac{d_{eq}\rho_v(U_{v,\text{eff}} + U_{L,\text{eff}})}{\mu_v}$ Calculate k'_L from penetration model (use time for liquid to flow distance s). $k'_L = 2(D_L U_{L,\text{eff}}/\pi S)^{1/2}$.	[T] Check of 132 data points showed average deviation 14.6% from theory. Johnstone and Pigford [Ref. 84] correlation (5-18-F) has exponent on N_{Re} rounded to 0.8. Assume gauze packing is completely wet. Thus, $a_{\text{eff}} = a_p$ to calculate H_G and H_L. Same approach may be used generally applicable to sheet-metal packings, but they will not be completely wet and need to estimate transfer area. L = liquid flux, kg/s m^2, G = vapor flux, kg/s m^2. Fit to data shown in Ref. [45]. $H_G = \dfrac{G}{k'_v a_p \rho_v}$, $H_L = \dfrac{L}{k'_L a_p \rho_L}$ effective velocities $U_{v,\text{eff}} = \dfrac{U_{v,\text{super}}}{\varepsilon \sin \theta}$, $U_{L,\text{eff}} = \dfrac{3\Gamma}{2\rho_L}\left(\dfrac{\rho_L^2 g}{3\mu_L\Gamma}\right)^{0.333}$, $\Gamma = \dfrac{L}{\text{Per}}$ $\text{Per} = \dfrac{\text{Perimeter}}{\text{Area}} = \dfrac{4S + 2B}{Bh}$	[45] [63] p. 310, 326 [149] p. 356, 362 [156]

TABLE 5-24   Mass-Transfer Correlations for Packed Two-Phase Contactors—Absorption, Distillation, Cooling Towers, and Extractors (Packing Is Inert) (*Concluded*)

Situation	Correlations	Comments E = Empirical, S = Semiempirical, T = Theoretical	References[*]
I. Distillation and absorption, counter-current flow. Structured packing with corrugations. Rocha, Bravo, and Fair correlation.	$N_{Sh,G} = \dfrac{k_g S}{D_g} = 0.054\, N_{Re}^{0.8}\, N_{Sc}^{0.33}$ $u_{v,\text{eff}} = \dfrac{u_{g,\text{super}}}{\varepsilon(1 - h_L)\sin\theta}, \quad u_{L,\text{eff}} = \dfrac{u_{\text{liq,super}}}{\varepsilon h_L \sin\theta},$ $k_L = 2\left(\dfrac{D_L C_E u_{L,\text{eff}}}{\pi S}\right)$ $H_{OG} = H_G + \lambda\, H_L = \dfrac{u_{g,\text{super}}}{k_g a_e} + \dfrac{\lambda u_{L,\text{super}}}{k_L a_e}$ Interfacial area: $\dfrac{a_e}{a_p} = F_{SE}\,\dfrac{29.12\,(N_{We}N_{Fr})^{0.15}\,S^{0.359}}{N_{Re,L}^{0.2}\,\varepsilon^{0.6}\,(1 - 0.93\cos\gamma)(\sin\theta)^{0.3}}$ Packing factors: |	a_p	ε	F_{SE}	θ | |---|---|---|---|---| | Flexi-pac 2 | 233 | 0.95 | 0.350 | 45° | | Gempak 2A | 233 | 0.95 | 0.344 | 45° | | Intalox 2T | 213 | 0.95 | 0.415 | 45° | | Mellapak 350Y | 350 | 0.93 | 0.350 | 45° |	[E, T] Modification of Bravo, Rocha, and Fair (5-24-H). Same definitions as in (5-24-H) unless defined differently here. Recommended [156]. h_L = fractional hold-up of liquid C_E = factor for slow surface renewal $C_E \sim 0.9$ a_e = effective area/volume (1/m) a_p = packing surface area/volume (1/m) F_{SE} = surface enhancement factor γ = contact angle; for sheet metal, $\cos\gamma = 0.9$ for $\sigma < 0.055$ N/m $\cos\gamma = 5.211 \times 10^{-16.8356}$, $\sigma > 0.055$ N/m $\lambda = \dfrac{m}{L/V}$, $m = \dfrac{dy}{dx}$ from equilibrium	[124], [156]
J. Rotating packed bed (Higee)	$\dfrac{k_L a_p}{D a_p}\left(1 - 0.93\,\dfrac{V_o}{V_t} - 1.13\,\dfrac{V_i}{V_t}\right) = 0.65\, N_{Sc}^{0.5}$ $\times\left(\dfrac{L}{a_p\mu}\right)^{0.17}\left(\dfrac{d_p^3 \rho^2 a_c}{\mu^2}\right)^{0.3}\left(\dfrac{L^2}{\rho a_p \sigma}\right)^{0.3}$ $500 \le N_{Sc} \le 1.2\,E5;\ 0.0023 \le L/(a_p\mu) \le 8.7$ $120 \le (d_p^3\rho^2 a_c)/\mu^2 \le 7.0\,E7;\ 3.7\,E-6 \le L^2/(\rho a_p\sigma) \le 9.4\,E-4$ $9.12 \le \dfrac{k_L a\, d_p}{D a_p} \le 2540$	[E] Studied oxygen desorption from water into N_2. Packing 0.22-mm-diameter stainless-steel mesh. $\varepsilon = 0.954$, $a_p = 829$ (1/m), $h_{\text{bed}} = 2$ cm. a = gas-liquid area/vol (1/m) L = liquid mass flux, kg/(m²S) a_c = centrifugal accel, m³/S V_i, V_o, V_t = volumes inside inner radius, between outer radius and housing, and total, respectively, m³. Coefficient (0.3) on centrifugal acceleration agrees with literature values (0.3–0.38).	[50]
K. High-voidage packings, cooling towers, splash-grid packings	$\dfrac{(Ka)_H V_{\text{tower}}}{L} = 0.07 + A'N'\left(\dfrac{L}{G_a}\right)^{-n'}$ A' and n' depend on deck type (Ref. 86), $0.060 \le A' \le 0.135$, $0.46 \le n' \le 0.62$. General form fits the graphical comparisons (Ref. 138).	[E] General form. G_a = lb dry air/hr ft². L = lb/h ft², N' = number of deck levels. $(Ka)_H$ = overall enthalpy transfer coefficient = lb/(h)(ft³)$\left(\dfrac{\text{lb water}}{\text{lb dry air}}\right)$ V_{tower} = tower volume, ft³/ft². If normal packings are used, use absorption mass-transfer correlations.	[86][104] p. 220 [138] p. 286
L. Liquid-liquid extraction, packed towers	Use k values for drops (Table 5-21). Enhancement due to packing is at most 20%.	[E] Packing decreases drop size and increases interfacial area.	[146] p. 79
M. Liquid-liquid extraction in rotating-disc contactor (RDC)	$\dfrac{k_{c,\text{RDC}}}{k_c} = 1.0 + 2.44\left(\dfrac{N}{N_{Cr}}\right)^{2.5}$ $N_{Cr} = 7.6 \times 10^{-4}\left(\dfrac{\sigma}{d_{\text{drop}}\,\mu_c}\right)\left(\dfrac{H}{D_{\text{tank}}}\right)$ $\dfrac{k_{d,\text{RDC}}}{k_d} = 1.0 + 1.825\left(\dfrac{N}{N_{Cr}}\right)\dfrac{H}{D_{\text{tank}}}$	k_c, k_d are for drops (Table 5-21) Breakage occurs when $N > N_{Cr}$. Maximum enhancement before breakage was factor of 2.0. N = impeller speed H = compartment height, D_{tank} = tank diameter, σ = interfacial tension, N/m. Done in 0.152 and 0.600 m RDC.	[36][146] p. 79
N. Liquid-liquid extraction, stirred tanks	See Table 5-22-E, F, G, and H.	[E]	

See also Sec. 14.
[*]See the beginning of the "Mass Transfer" subsection for references.